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Effects of Self-Explanation on the Expansion and Indulgent of Mathematics Teaching: Case of the Students of the University of Wachemo

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Abstract: *The main goal of this study is to investigate the effects of self-explanation in improving proof comprehension in mathematics instruction: the case of Wachemo university undergraduate mathematics students of academic year 2020/2021. Moreover it is intended to find out interconnection between students achievement in mathematics and self-explanation in proof comprehension. To assess student's self-explanation ability we applied self-explanation test and attitude test at the beginning and at the end to assess student's character change to ward mathematics. In this study there are 48 students participated in both control and experimental groups. This study showed that the effectiveness of self-explanation in proof comprehension in mathematics courses. This also leading to recommend for curriculum developers to consider in self-explanation. One of the most strong findings of research is that conceptual indulgent is an important elements of expertise hence self-explanation is the tool to enhance students conceptual understanding, along with realistic knowledge and technical facility. The alliance of realistic knowledge, procedural expertise, and conceptual indulgent makes all three components operational in powerful ways. So that self-explanation has prominent effect in supporting students to achieve the intended objectives in proving theorems. Of these, two areas were measured quantitatively: student achievement in calculus and transformation geometry, and self-concept toward these courses. Those quantitative research findings are presented in chapter four with the details of the data collection procedures, the quantitative results, and an analysis of the outcomes.*

Keywords: Self-effects, Mathematics Instruction, Comprehension, Indulgent.

1. Introduction

1.1. Background of the Study

Mathematics is an indispensable part of most of the degree disciplines. Even though, most studentscome to university inadequately prepared for it, and to struggle with, the mathematical conceptions on their course. The use of mathematics and mathematical skills is fundamental to almost any profession which is chosen by an individual. Nevertheless, when an individual decide on complete a particular degree program, they are often uninformed of the volume of mathematical content in their chosen discipline. As a result, students come to university inadequately, and those students who challenged to manage their mathematical understanding are usually more likely, inclined to dropping out. Undeniably, most of these students suffer from mathematics anxiety due to their past bad exposure with mathematics [1].

In the learning of mathematics, it is significant to note that students learn in various ways. Some students learn best when working in group while other students favor to work alone. No matter what, as instructors, we must be aware of the students that we are schooling. We must find every vital way to be effective in our method of teaching. Some instructors lecture their students and they do not give their students a chance to display their learning outside of coursework's. [2].

However, very little researches on how undergraduate students read proofs with the intent of learning mathematics from them. In a work to improve students' indulgent of proof, Weber and Mejia-Ramos established five proof-reading approaches that undergraduate Students can use to recover their proof comprehensions, which form the foundation for this study [3].

According to cognitive load theory, generating self-explanation requires high cognitive capacity by requiring that learners monitor their understanding and represent incoming information at the same time [4].

Students challenged with mathematical proofs, not only do they find it difficult to build proofs but it has also been shown they have difficulty in understanding it. Because they misapprehend a theorems as well as concepts then, therefore, misapply it [5]. It is important for students to obtain the abilities required to understand mathematical assertions since such assertions play a vital role in explaining the mathematics that helps us to better realize the world we live in. Additionally, the talents required to cognize mathematical proofs help students to "... think more clearly and effectively about mathematics" [6]. For that reason it is essential to perform more researches in the field of mathematics education that aims to improve the way proofs are imparted, assembled and understood. The study made on proof activities in the field of mathematics education has lean towards to focus more on proof structure tasks more willingly than proof comprehension tasks and yet one might dispute that in order for one to be able to construct mathematical proofs, one needs to understand previously proven statement and how these proofs are constructed first. For example, one needs to know the different types of proofs how to be construct and also understand how, and when, these different types of proof should be used to prove a theorem. But, it is somewhat surprising that there is comparatively less research into proof comprehension than proof construction. It is possible note that the majority of studies in the field of proof comprehension is not supported by realistic evidence because evaluating the proof comprehension skills of students successfully and perfectly could be considered challenging.

1.2. Statement of the Problem

One of the foremost goals of mathematics education is to cultivate students' problem-solving abilities. However, current mathematics education programs have been criticized for not meeting such demand in this era of the knowledge economy. Traditionally, mathematics classes in secondary school as well as tertiary level; students were taught using a lecture teaching method. Therefore, the aim of this research is to determine the usefulness of a problem-solving teaching strategy on the learning outcomes. Mathematics is beyond knowing, but it is also about doing. As educationalists, we confess the importance of understanding the ultimate concepts that strengthen mathematics. Yet the assessments of our students are often system based. Teachers use both theoretical and practical methods of instruction when teaching students to solve problems necessitating algebraic cognitive. Successfully balancing theoretical and practical emphases in classroom instruction support students as they begin to develop the algebra skills needed for success in University mathematics classes. Having these suppositions, the following research questions were investigated.

1. To what extent self-explanation improve comprehension?
2. Can we help students to read more effectively?
3. To what extent self-explanation change underlying reading behavior?
4. To what extent self-explanation method of instruction works in a usual pedagogical setting?
5. How do students read proofs?

Hypothesis of this study:-

The following null hypothesis was tested:

1) There is no statistically significant relationship between students" Proof comprehension and self-explanation understanding
2) There is no significant effects between the two methods of teachings (treatment based versus conventional) on the population means of the first and second year undergraduate mathematics students' scores on the post implementation of self-explanation test Transformation geometry and calculus courses.

1.3. **Objective**

General objective of this paper is to examine effects of self-explanation in improving proof comprehension in mathematics instruction with respect to algebraic and geometric concepts.

Specifically this research will be aiming to investigate:-

1) To examine the effects of different self-explanation stimulates on theoretical understanding and improving problem solving enactment.
2) To examine whether students provided with an open self-explanation stimulate exerted more cognitive efforts while engendering explanations and examining problems.
3) To investigate the quality of explanations elicited from different stimuli.

2. LITERATURE REVIEW

2.1. Mathematical Understanding

The literature review on proof conception is reasonably few. As literature review shows that studies on proof assessment indicate that mathematicians do not unavoidably evaluate their students' understanding of a given proof effectively [7]. For instance, maintain that mathematicians' ways of testing their students' understanding of a proof usually involve nothing outside recalling the statements and its proof also conceded this. As indicated in some research works on students understanding of proofs in a particular task. In the works of study experts in the field of mathematics reported that they measured their students' understanding of proofs by asking students to construct a proof for a analogous theorem to the one that was verified in class, and/or asking them to imitate a proof; and some said they do not assess their students' understanding of a proof maintain that students can pass simply by remembering the statement and proof of each theorem as offered in class;

this, however, as they draw attention to that, does not effectively reflect students' understanding of mathematical concepts [7].

2.2. Self-explanation

According to most of educational researches; self-explanations can support students develop their abstract and practical knowledge of mathematics by assimilating knowledge of the problem solving skill and knowledge of the essential principles in the learner's intellectual cognitions[8]. While there are some findings that support the previously stated hypotheses of the potential benefits of self-explanation in mathematics, the actual empirical findings are uncertain. There are studies that reveal a positive effect of self-explanation in mathematics however, there are also studies that reveal no effect or a negative effect for self-explanations [9-13]. However, there are still a need to determine even if self-explanation in mathematics has actually been shown to be useful in past researches.

2.3. Explanation and Evidence in Mathematics Education

As we all know that explanations and evidences are fundamental to the field of mathematics. For that reason, several prominent mathematics experts, researchers and organizations in mathematics education have argued that explanations and evidences should also play an essential part in all mathematics classrooms [6, 14, 15]. There are enormous literature in this area of specialty on explanation and evidences; reviews of this literature have been presented by Harel, G. [16] and Sowder, L. [17]; Marriotti, M., Weber, K. Yackel, E. [18] and Hanna, G [19].

2.4. Conceptual Understanding

According to some research publication regarding to students transitioning from arithmetic to algebra often contains full of challenges with misconceptions. Recently the use of concrete models in teaching solving equations has become a more common practice to help students develop conceptual understanding of equality. In the majority of prior experiences, the *equals* sign was active. In algebra, students must see the *equals* sign as relational, denoting either side has equal value. Students as early as third grade can conceive of this aspect of equality when they are given experiences that feature the *equals* sign in situations that allow students to recognize quantitative sameness [20]. Too often, children do not have such experiences with equality until formal algebra study.

Although there has been some argument over the relationship between theoretical and practical knowledge and which type of understanding develops first as students encounter new mathematics [14]. As it was proposed; a mediating viewpoint, that, in fact, the two types of knowledge are not necessarily distinct, but rather opposite ends of a continuum and enlargements in one type of understanding typically result in enhancements in the other type [22].

2.5. Multiple Representations

In the sympathetic characteristics of the internalization in the process of intellectual representation, we observe to the Vygotskian conception of mediation. L. Vygotsky and his coworkes argue that determination of individual cognition shall be presented by the following scheme: communal (social) activity – culture signs/ symbols – singular activity – singular cognition [23]. Vygotsky argue that "Every function in the child's cultural growth appears two times: first, on the social level, and then, on the individual level" [23]. The importance of the method of cognitive symmetry is that it gives a clue for designing the structure of externalization process based on the scheme of the internalization one. In addition to, if internalization aims at understanding (e.g., seeing, comprehension, interpretation, etc.), externalization tends toward creativity (e.g., construction, generalization, abstraction, etc.).

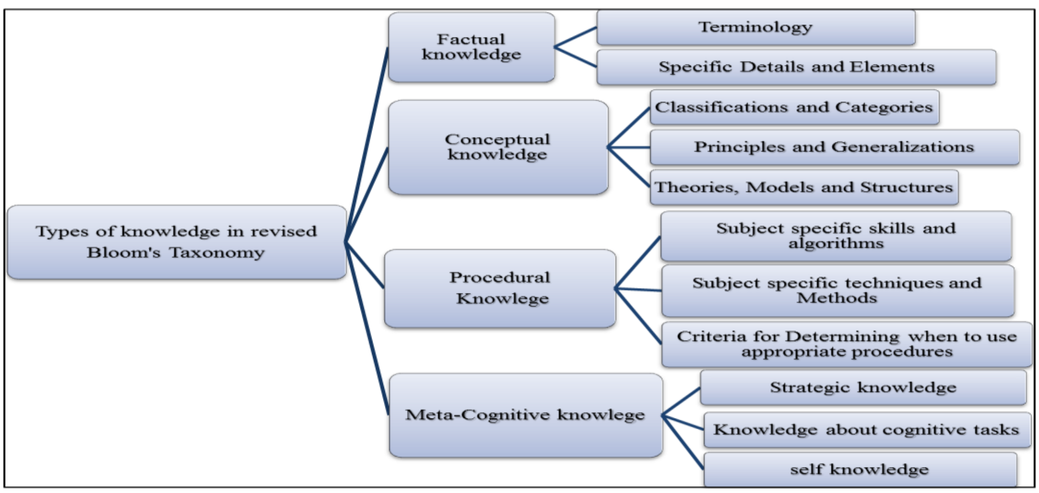

Figure 1. *Types of knowledge in revised Bloom's Taxonomy.*

There are many researches discussed about the meaning of concept of representation. We define this concept benefit from all of this study; representation is a mental form of a mathematical theory and to present this theory in to various form [14]. This representation can be form of sometimes a tableau, sometimes an equation, sometimes a concrete material, sometimes a sign or symbol, sometimes a formula and sometimes a figure or picture etc. [24, 27].

It is to say that representations act below situations;

1) To form developed mental senses,
2) To explain people themselves, to communicate with one another person and to make procedures,
3) To produce knowledge.

It has been demanded that disclosure to multiple representations leads to deeper theoretical and practical understanding. For instance, Bloom's suggests that "the cognitive linking of representations creates a whole that is more than the sum of its parts [28]. It enables us to 'see' complex ideas in a new way and apply them more effectively". In this paper, 'deeper understanding' should be considered in terms of using multiple external representations to indorse abstraction, to inspire generalization and to teach the relation between representations. The representation of algebra often involves the translation from verbal information into symbolic expression then into equation [29]. Yerushalmy, M. and Shtenberg [21] state that forming situations before using symbolic representations for the functions will help students better understand algebraic concepts and functions.

3. RESEARCH METHODOLOGY

3.1. Research Methodology

The purpose of this section is to present the anticipated procedure for this study. It include description of the research design, the population and its sample, the description of the variable, the data collection instrument/tools and the data analysis approach used to address each research question.

3.2. Method

The experimental research method was used in the study to achieve the research purposes. The subjects will be wachemo university mathematics department students and the experimental period will be three months. Due to the nature of this research, random sampling will be utilized for the study. Under this condition, the experimental research design will use to obtain adequate control of sources of invalidity, and equivalent control group design to each subject to groups.

Table 1. *Pre-test post test matrix.*

Group	Pre-test	Treatment	Post-test
Experimental	X_1	X	X_2
Control	X_3		X_4

3.3. Methods of Data Analysis and Presentation

Qualitative and quantitative analysis of data proposed by the researcher was used. To present the collected data tables will be used. Depending on the nature of data appropriate descriptive and inferential statistical tools will be applied. The descriptive statistical tools proposed are the mean and standard deviation. The inferential statistical tools proposed for this study is correlations, ANOVA.

The population of this study were undergraduate students who have enrolled for mathematics. All participants considered were from Wachemo University. We took the survey items in Weber, K. and Mejia-Ramos, J. P. [3] study where they asked mathematics majors to indicate the extent to which the abovementioned proof-reading schemes were insightful of their own. All undergraduate students (48) who is enrolled in the department of mathematics completed the survey. All participants asked to indicate their choice using a five-point Likert scale (strongly agree (5), agree (4), neutral (3), disagree (2), and strongly disagree (1)). We used the statistical software SPSS of the recent version to determine if there was a statistically significant difference between the two groups. We presentes our findings in the next section.

This research took the whole undergraduate students whom enrolled for transformation geometry and CalculusI in Wachemo University. Among those, students were selected purposively depending on their previous semester's achievement. The two groups were unsystematically assigned as either the experimental group or the control group. There were 21 students in the experimental group and 27 students in the control group. These two groups were pre-tested, administered a treatment, and then post-tested. The distribution of the formal sample.

3.4. Method of Data Analysis and Presentation

Quantitative and qualitative analysis of data anticipated by the researcher were used. To present the collected data tables was used.

3.5. Participants

The students in targeted group that were examined in this study during the 2020/2021 academic years.

3.6. Data Collection

During March 2021, parallel forms of the calculus and transformation geometry test, containing similar content with their curriculum presented for pretest participants. A posttest identical to the pretest was given to students at the end of the self-explanation training. In addition to these data, Self-explanation test was examined in seriousness for its degree of proof comprehension emphases.

3.7. Methods of Data Analysis and Presentation

Quantitative and qualitative analysis of data proposed by the researcher were used. To present the collected data tables were used. Depending on the nature of data appropriate descriptive and inferential statistical tools were applied. The descriptive statistical tools proposed were the mean and standard deviation. The inferential statistical tools proposed for this study is correlations and ANOVA..

4. RESULTS AND DISCUSSION

The results of the hypothesis are presented in this section. ANOVA are used for testing the hypothesis at a significant level of 0.05. All the statistical analyses are carried out by using SPSS version 20.0 for windows.

As we have mentioned earlier the goal of this study is to evaluate the effects of self-explanation in improving proof indulgent in mathematics instruction in the process of teaching-learning of mathematics. Of these, two areas were measured quantitatively: students' achievement in calculus and transformation, and self-concept toward mathematics. These quantitative research findings are presented in this chapter with the details of the data collection procedures, the quantitative results, and an analysis of the findings.

4.1. Descriptive Statistics

Descriptive statistics presented in this study is used to identify means, and standard deviations, for the two groups are summarized as there regards.

4.1.1. Mathematics Self-concept Test for Proof Reading Strategies

We adopted the survey items in this study to ask mathematics majors to indicate the extent to which the aforesaid proof-reading strategies are reflective of their own [24]. Forty two undergraduate students who were enrolled in the department of mathematics completed the survey. All participants were asked to indicate their choice using a five-point Likert scale (strongly agree (5), agree (4), neutral (3), disagree (2), and strongly disagree (1)).

(80-100)% -------- Strongly agree
(50-59)% -------- neutral
(60-79)% -------- Agree
(40-49)% -------- disagree
Below 40% --------strongly disagree

Table 2. *Descriptive statistics related to the scores of Self-concept test for the groups.*

Name of Group	N		Self-concept Pretest	Self-concept posttest
Experimental group	21	Mean	79.048	87.62
		Std. D	11.7918	9.9522
Control group	27	Mean	80.74	78.60
		Std. D	10.35	113.5032

Self-concept pretest Self-concept posttest * Name of Group.

As the result showed in above table for an instrument Self-concept test; the mean scores of the experimental group is statistically significant higher than the Control group mean scores.

4.1.2. Self –explanation Test

Self-explanation achievement test is used to measure students' ability read and explain the concepts and theorems in the given courses. This test consists of 8 items addressing key learning goals specified in the Mathematics Curriculum for second and first year mathematics major students of Wachemo University.

Table 3. *Descriptive statistics related to the scores of Self-explanation test for the groups.*

Group	N		Self-explanation post-test	Self-explanation pre-test
Experimental	21	Mean	4.2143	2.7619
		Std.	2.22807	2.02866
Control	27	Mean	2.2778	2.1852
		Std.	1.94804	1.62994

As the result showed in Table 3 for an instrument self-explanation achievement test the mean scores of the experimental group is higher than that of the control group. When the mean scores from the pre administration and post administrations of the instruments, are compared, the EG showed an increase from 2.7619 to 4.2143. The result in this table also is supported by the following clustered box plot.

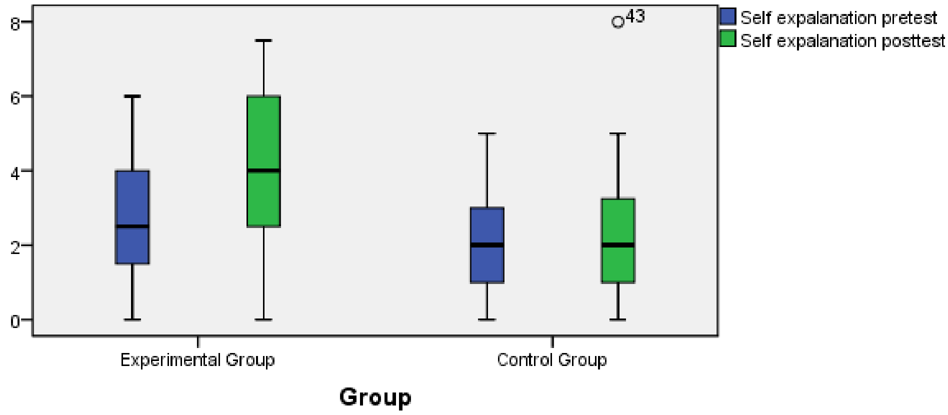

Figure 2. *Clustered box plot for Conceptual pretest and Conceptual posttest of experimental group and control group.*

Above figure indicates, the median scores of Conceptual posttest for experimental group is meaningfully greater than that of control group.

4.1.3. Procedural Understanding in Proof Reading Test

Procedural proof reading test is aimed to measure student's capability to perform on the given problems with respect to solving and representing a given problem with multiple representation technique to come with the solutions and justification by applying the specific procedure. This test consists of 8 items addressing the learning goals specified in the mathematics.

Table 4. *Descriptive statistics related to procedural understanding.*

Group	**N**		**Procedural posttest**	**Procedural pretest**
Experimental group	21	Mean	4.38	3.95
		Std.	0.498	0.590
Control group	27	Mean	4.130	4.00
		Std.	0.67516	0.5175

As the result showed in Table 4 for an instrument procedural proof reading test the mean scores of the experimental group is higher than that of the control group. When the mean scores from the pre administration and post administrations of the instruments, are compared, the EG showed an increase from 3.95 to 4.38. The result in this table also is supported by the following clustered bar graph.

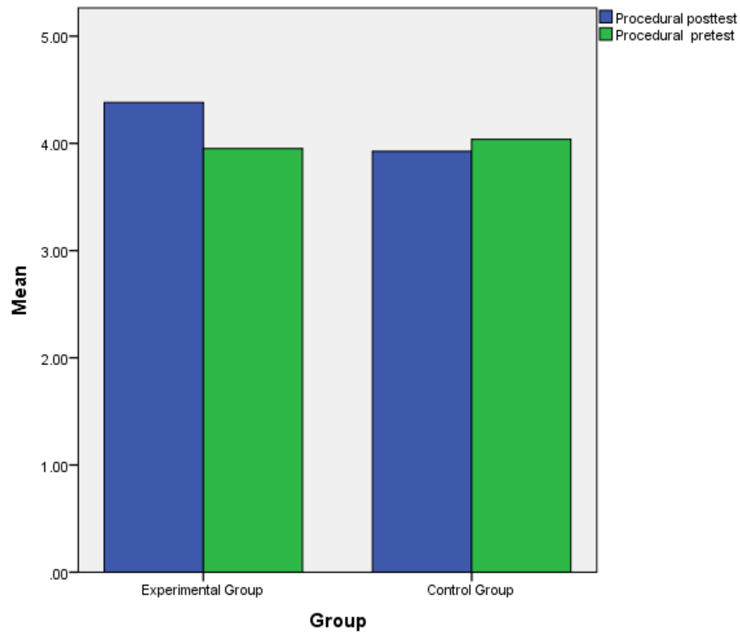

Figure 3. *Clustered Bar graph.*

Figure shows that, the mean scores of procedural posttest for experimental group is higher greater than that of control group.

4.2. Inferential Statistics

Findings of the study related to the hypotheses are presented in this section. Hypotheses related to the research question are:

1) *Null hypothesis 1:* There is no statistically significant relationship between students" Proof comprehension and self-explanation indulgent. To test this hypothesis the statistical technique of ANCOVA was used.

Table 5. *Test of Between-Subjects effects.*

Source	Dependent Variable	SS	df	MS	F	Sig.	Partial Eta Squared	Observed Power
Corrected Model	Self-explanation pretest	4.907	3	1.636	.478	.699	.032	.139
	Self-explanation posttest	78.402	3	26.134	7.018	.001	.324	.970
Intercept	Self-explanation pretest	6.878	1	6.878	2.012	.163	.044	.284
	Self-explanation posttest	59.688	1	59.688	16.029	.000	.267	.975
Procedural posttest	Self-explanation pretest	.556	1	.556	.163	.689	.004	.068
	Self-explanation posttest	26.086	1	26.086	0.005	.011	.137	.735
Procedural Pretest	Self-explanation pretest	.447	1	.447	.131	.719	.003	.064
	Self-explanation posttest	8.785	1	8.785	2.359	.132	.051	.324
Group	Self-explanation pretest	4.230	1	4.230	1.237	.272	.027	.193
	Self-explanation posttest	64.215	1	64.215	17.244	.000	.282	.982
Error	Self-explanation pretest	150.406	44	3.418				
	Self-explanation posttest	163.848	44	3.724				

Total	Self-explanation pretest	440.500	48
	Self-explanation posttest	711.000	48
Corrected Total	Self-explanation pretest	155.313	47
	Self-explanation posttest	242.250	47

Computed using alpha = 0.05

As it was shown in Table, it can be said that theoretical understanding based instruction has a significant effect on the dependent variable posttest scores of the Practical understanding test [F (1, 44) = .005, p = .011]. So null hypothesis is rejected.

2) *Null hypothesis2*: There is no significant effects of two methods of instruction such as (treatment based versus conventional) on the population means of the Undergraduate mathematics students' scores on the post implementation of conceptual understanding and procedural posttest. To test this hypothesis the statistical technique of MANOVA is used. The following table summarizes the result for the MANOVA.

Table 6. *Multivariate test results.*

Effect	Wilks' Lambda (λ)	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Observed Power
Intercept	0.731	7.921	2.000	43.000	0.001	0.269	0.940
Group	0.710	8.774	2.000	43.000	0.001	0.290	0.960

Design: Intercept + Group.
Computed using alpha = 0.05.

According to the results displayed in above table, using the Wilks' Lambda test, significant main effects are detected between the groups (λ = 0.710, p = .001). This means that statistically significant differences are identified between the treatments based instruction and conventional instruction on the collective dependent variables of the conceptual posttest, and procedural posttest. Therefore, the first null hypothesis is rejected. In other words, self-explanation focused instructions have an effect on the collective dependent variables of the conceptual posttest and procedural posttest as compared to conventional teaching method.

Table 7. *Correlation coefficient Matrix.*

	Self concept posttest	Self concept pretest	Self expalanation posttest	Self expalanation pretest
Self concept posttest	1	-.061	-.121	-.021
Self concept pretest	-.061	1	-.215	-.064
Self expalanation posttest	-.121	-.215	1	.461**
Self expalanation pretest	-.021	-.064	.461**	1

**. Correlation is significant at the 0.01 level (2-tailed).

From table 7, we can see that the correlation coefficient at 0.01 significant level for all the instruments and Group (experimental and control); there is a strong correlation for the post tests. And also we can see that between the instruments there is strong relation for example the correlation coefficient for self-explanation post-test and self-explanation pretest is 0.461 which is more than 0.01 as this value indicate there is strong correlation between the two instruments.

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

In this chapter, the conclusion of this study, in the light of the theoretical and empirical findings obtained from the research literature relating with the effects of self-explanation is discussed. The implications of the findings of the present study, the internal and external validity of the present research design, and the suggestions for future research about effects of self-explanation in transformation geometry and calculus are also given in this chapter. This chapter summarizes research questions and literature presented in chapters 1 and 2 with the methods and results presented and discussed in chapters 3 and 4.

5.1. Summary

The main aim of this study is to assess the self-explanation in improving proof comprehension in mathematics instruction with respect to proof comprehension of mathematics major students in Wachemo University. The summary part was presented with respect to each variable in this study.

5.1.1. Relationship Between Self-explanation with Procedural Knowledge

The result of this research as it was shown in Table from null hypothesis1, it is possible to say that conceptual indulgent based instruction has a significant effect on the dependent variable posttest scores of the Procedural understanding test [F (1, 44) = 0.005, p = .011]. So null hypothesis is also rejected.

The effect of the treatment based on conceptual and procedural understanding was investigated by the first to fourth research questions which were a quantitative one. By posing this question, the focus was given on finding a significant difference between the experimental and control groups in terms of the conceptual and procedural knowledge on the effects of self-explanation in proof comprehension and Conceptual and procedural calculus and transformation geometry test.

According to Schoenfeld, A. H. [15] students should learn the conventional representational modes to improve their mathematical reasoning, however; idiosyncratic (personal) representations which are specific to certain problems and belong to the individual should also be valued in the mathematics classrooms in order to establish a conceptual link between these modes of representations and conventional ones. Furthermore, the results from the literature review Hanna, G. [6] and Leont'ev, A. [23] in conjunction with the results of this study indicate that the use of emphasises for conceptual and procedural knowledge gives the opportunity of making translations among representational modes.

5.1.2. Interconnection Between a Performance in Class Achievement, Conceptual and Procedural Knowledge

As we have seen from the result specifically form hypothesis 2 also from correlation coefficient there is strong interdependence between student performance with knowledge of conceptual and procedural understanding.

One of the most strong findings of research is that conceptual understanding is an important component of proficiency hence self-explanation is the tool to enhance students conceptual, along with factual knowledge and procedural facility. The alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways. So that self-explanation has prominent effect in supporting students to achieve the intended objectives in proving theorems.

5.1.3. Effects of Emphasis on Self-explanation in Proving Theorems on Student’s Ability to Model Equation

From the inferential data interpretation which presented in tables we can collectively see that the mean score of students in experimental group is showed higher mean score than control group in both conceptual and procedural understanding. This clearly indicates that there is an impact of the treatment.

5.2. Conclusions

Based on the findings of the study, the following conclusions were drawn:

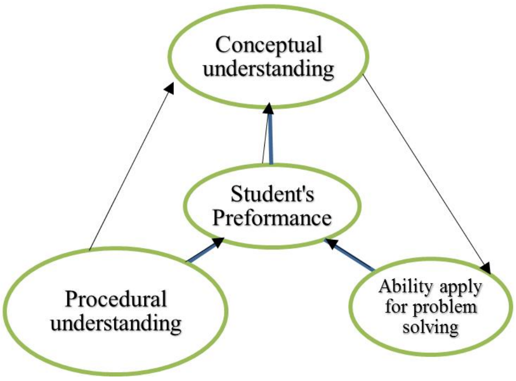

Figure 4. *Interconnection of knowledge.*

1) Self-explanation focused instructions gave a significant scientific acquisition in understanding proof reading.
2) Self-explanation focused instruction is able to give more scientific knowledge and skills to reduce the difficulties in proving theorems than that of conventional teaching method.
3) Self-explanation focused based instruction gave positive self-concept towards mathematic as course than that of conventional instruction.
4) Emphasis on self-explanation focused has a promising impact on student's ability to model equation to multiple representations as compared to conventional method of classroom instructions.

5.3. Recommendations

Generally, based on the results, the researcher recommends the following:

i. All of the data for this study was collected from students only so, future researcher should extend their focuses on the data from students and their teachers, because teachers have also impact on shaping students' cognitive domain.

ii. Further studies might also be conducted beyond the transformation geometry and calculus courses. And multiple representation-based approaches should be implemented to every topic in mathematics.

iii. This study was carried out in one month time. Therefore, further research should be applied on multiple representations-based approaches for longer periods of time, and incorporating more chapters. If the treatment would last longer and if it includes ample topic, a better chance to gain evidence on students` mathematical learning using self-explanation method.

iv. Finally, this study did not include representational modes like multimedia instructional technology to see impact of conceptual and procedural knowledge so the researcher recommends further research on the effects of multimedia instructional modes on the developing students understanding on the algebraic situation.

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List of Abbreviations

MANCOVA	Multivariate analysis of covariance
ANOVA	Analysis of variance
ANCOVA	Analysis of covariance
Sig	Significance
df	Degree of freedom
N	Sample size
MS	Mean square
P	Significance level
F	F statistics
SS	Sum of squares
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council

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