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PACE Technique to Error Categories: Its Effectiveness in Improving Word Problem Performance in Mathematics among the 7th Grade Students

Roghene A. Coloma[1], Florante P. Ibarra[2], Amirul Mukminin[3], Lenny Marzulina[4] Herri Mulyono[5],Kasinyo Harto[6]
[1]College of Education, Central Luzon State University, Science City of Munoz, Nueva Ecija, Philippines
[2]College of Education, Central Luzon State University, Science City of Munoz, Nueva Ecija, Philippines, corresponding e-mail: fpibarra@clsu.edu.ph
[3]Faculty of Education, Universitas Jambi, Indonesia
[4,6] Universitas Islam Negeri Raden Fatah, Palembang, Indonesia
[5]University of Muhammadiyah Prof. Dr. Hamka, Jakarta, Indonesia
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Abstract
This research determined the effectiveness of the PACE (present the error, analyze the error, correct the error, explain the error) technique in improving the mathematics performance in solving word problems among 7th Grade students. The study utilized quasi-experimental research design with a validated researcher-made 7th Grade Mathematics test as the main instrument. The test was administered to 38 students before and after the implementation of the PACE technique. The data collected were treated using frequency counts, means, rates, Newman's error analysis tool and t-test for dependent means. Before the implementation of the PACE technique, the students had a fair performance in solving word problems. In terms of their errors, transformation was the major error category in proportional relationships, reasoning about equations and inequalities and geometric and measurement reasoning; comprehension in reasoning with rational numbers, probabilistic reasoning and reasoning about population samples and comparing populations; and processing in reasoning about expressions. On the other hand, the students got an outstanding performance after the implementation of the PACE technique. After using the PACE technique, majority of the students had no errors in proportional relationships, reasoning with rational numbers, reasoning about expressions and reasoning about equations and inequalities; encoding errors in probabilistic reasoning and geometric and measurement reasoning; and transformation errors in reasoning about population samples and comparing populations. There was also a significant difference between the level of performance of students before and after using the PACE technique. Reading, comprehension, transformation and encoding are the error categories the PACE technique was most appropriate. And lastly, the PACE-Based 7th Grade Mathematics Workbook is very highly valid.

Keywords: error analysis, error categories, PACE technique, PACE-based workbook.

1. Introduction

Low performance of students in mathematics word problem solving can be disappointing and alarming for students, teachers, school administrators and parents which PACE (Present the error, Analyse the error, Correct the error and Explain the error) technique purports to address. This technique is very similar to the Error Bull's-Eye and Solution Inspector techniques of Ragma (2014) which are both in his instructional intervention plan. In using this technique, students apply, develop and enhance their higher order thinking skills whenever prompted with a word problem since they are required to analyze, correct and explain an erroneous calculation that is presented. This technique also reduces the errors of students in dealing with a mathematical problem which will eventually improve their performance in mathematics.

It is really critical and challenging to increase the academic performance of students with an emphasis on improving the performance of low-achieving students (Muhaimin et al., 2020; Prasojo et al., 2020). Comparing the mathematics performance of American students with the performance of other students in other countries through Trends in International Mathematics and Science Study (Mullis et al., 2012) and Program for International Student Assessment (Kastberg et al., 2016) gave a realization that American students are really lagging behind. In connection, the U.S. Department of Education released the results from the 2017 National Assessment of Educational Progress (NAEP) and show that the average score of eighth-grade students in North Carolina was 284; 64 percent of them are categorized as not proficient. These data indicate that the students in the state are not performing and meeting the expected standards. This needs to be addressed as soon as possible otherwise they continue to struggle and this may affect their future.

Further, other Mathematics teachers were interviewed and found out that students have low performance in the subject (Rakimahwati et al., 2022). As observed, many students in this subject are failing due to the lack of practice and mastery in Mathematics. For this school year 2019-2020, the results of the 1st and 2nd Quarter Unit Assessments have an overall passing rate of 48 percent and 44 percent respectively. In addition, based on their End of Grade (EOG) released results, more than 50 percent of them received a letter grade of Not Proficient (NP). It is also sad to note that the school received a School Performance Grade of D and a growth designation of “Did Not Meet” for the 2018-2019 school year based on the North Carolina Schools Report Card. These data indicate that the students generally have low performance. According to Zamzam and Patricia (2018), problem solving plays an important role in mathematics education because it could train students to be mathematically-minded. The focus is always on processing when teaching Mathematics. For the students to successfully learn to solve mathematical problems and eventually increase their performance, they really need to acquire mathematical processing skills one of which is problem solving.

Encountering mathematical errors is common and natural on the part of the students especially when learning Mathematics. The errors of students are often brought by their low interest in the subject, high anxiety, negative attitude and mindset, lack of recall, misconception and misinterpretation, poor mastery and carelessness. The dismal performance of students is probably owing to their weaknesses in understanding the given worded problems, extracting clues, hints and ideas from the problem, recalling the topics and formulas that they need to answer the problem, solving the problems with correct operations and simplifying and presenting their final answer. The foregoing scenarios prompted the researchers to implement the PACE Technique and the success was assumed to lessen the errors of students in solving word problems and improve their performance in Mathematics.

There are so many researchers ventured studies related to error analysis in mathematics and who made use of Newman’s (1977) procedures (Abdullah, 2015; Alhassora, 2017; Hariyani, 2018; Kurniati et al, 2021; Rohmah, 2018; Rosli, 2016; Saleh et al., 2017; Zamzam & Patricia, 2018). All these researchers agreed to what Newman had discovered when they have also found out that when students attempt mathematical questions, it is probable for them to commit errors during the process or even in presenting their final answers and it is often difficult to know whether they have a problem with their processing, or whether they made a mistake at some other point.

This study was rooted from the Theory of Errors and Error Categories of Newman (1977) which explicates that student usually encounter errors in reading, comprehension, transformation, processing and encoding in any of the stages of word problem solving. Newman (1977) elucidated that it is very common for a person to encounter issues, problems and difficulties when confronted with mathematical questions. In this case, he may incur errors in reading, comprehension, transformation, processing and encoding. Students normally commit varied mistakes in any of the stages. In is in this light that Newman suggested and devised a series of prompts to help teachers determine where students are making mistakes in problem solving. The prompts that were included were “Please read the question to me. If you don’t know a word, leave it out.”, “Tell me what the question is asking you to do.”, “Tell me how you are going to find the answer.”, “Show me what to do to get the answer. Talk aloud as you do it so that I can understand how you are thinking.” and “Now write the answer to your question.”

These prompts are a perfect opportunity for the students to look back and check what the question was asking them to do. This also solves their problem on encountering silly mistakes of using amazing strategies and reasoning but leaving out a small and vital piece of information right at the end. Newman (1977) realised that when a student makes a mistake in problem solving, it is not necessarily because of flaws in their processing skills. Students could also have made an error in the reading or interpretation of the question, the identification of which strategies to employ, or in the final communication of their answer. By using Newman’s prompts, one can identify where his students are going wrong in their problem solving, so that he can accurately assess where they need extra assistance. The regular use of the prompts will also give his students a good framework for solving problems and will give them great opportunities to both understand and use mathematical language more effectively.

The research studies on error analysis (Alhassora, 2017; Hariyani, 2018; Kurniati et al, 2021; Rosli, 2016; Rohmah, 2018; Saleh et al, 2017; Yugdu, 2020; Zamzam & Patrcicia, 2018) showed that students are probable to encounter mistakes when they deal with mathematical questions. They normally incur errors in reading, comprehension, transformation, processing and encoding in any of the stages in word problem solving. Yugdu (2020) specified that the different error categories and their causes can be addressed through varied instructional interventions. Reading errors can be avoided by providing motivational instructional activities, games and differentiated instruction. Comprehension errors can be limited by concept attainment and processing. Transformation errors can also be minimized through direct instruction, memory-bank game and the think-pair-share activities. Processing errors can be lessened thru error targeting and correcting and explicit instruction. And lastly, encoding errors can be reduced by solve-and-compare, cooperative learning, group activities, and check-on-me activities. It is expected that students can perform better in Mathematics if all error categories in each problem-solving stage with their respective causes if all the aforementioned concepts and strategies are considered. The low performance and the different error categories of students and their causes can be addressed through varied instructional interventions. That is why the PACE technique and the 7th Grade PACE-Based Mathematics Workbook are both instructional interventions that would opt to solve the issues of the students.

2. Methods and materials

The study utilized a quasi-experimental research design. This research design is an empirical interventional study used to estimate the casual impact of an intervention on target population without random assignment (Cook & Campbell, 1979). Purposive sampling was utilized as the method in choosing the subjects of this study. This is also known as judgment, selective or subjective sampling. It is a sampling technique in which the researcher relies on his own judgment when choosing members of population to participate in the study (Lewin, 2019). For the purpose of this study, the researchers purposefully selected all 38 students in Mathematics classes based on their content mastery in the learning process.

The first stage of the study was to conduct the pre-test using a researcher-made test. The students were given one (1) hour to complete the pre-test and permitted to use a scientific calculator. The second stage of the study was to implement the PACE Technique for a month. After its implementation, the post-test was conducted; this was the third stage. The same researcher-made test was used. For the students not to see a very familiar test, the questions were rearranged. The students were also given one (1) hour to complete the post-test and permitted to use a scientific calculator.

To determine the level of performance of students in solving word problems before and after using the PACE Technique along the seven (7) standards, frequency counts, means and rates were utilized. To determine the error categories of students before and after using the PACE technique in terms of reading, comprehension, transformation, process and encoding the Newman Error Analysis Tool (1977) was used. The t-test for dependent means assuming normal distribution was utilized to compare the significant difference between the level of performance and the number of errors of students in solving word problems before and after using the PACE Technique.

3. Results and Discussion

3.1 Students Level of Performance in Solving Word Problems Before the Implementation of PACE Technique

Table 1 shows the level of performance of students in solving word problems before the implementation of the PACE technique. It can be gleaned from the table that the students had an overall mean score of 2.16 which is interpreted as *fair performance*. This most likely implies that the students did not achieve the expected standards and competencies in the subject. This can be correlated to the fact that all the items were word problems that really required higher-order thinking and mathematical skills.

Moreover, the students scored the highest in reasoning about expressions with a mean score of 2.79, interpreted as *fair performance.* It can be understood that the students tend to perform at the average level in this topic. On the other hand, they scored the lowest in reasoning about population samples and comparing populations with a mean score of 1.45 which means *poor performance* suggesting that they had not obtained the expected competencies in this topic. This topic comprises the greatest number of sub-topics and students really need to learn the basics of measures of centre before mastering measures of variability which the given question is related to.

Table 1. Level of performance of students in solving word problems before the implementation of the PACE technique

Standards/Topics	Mean Score	Descriptive Equivalent
Proportional Relationships	2.39	Fair Performance (FP)
Reasoning with Rational Numbers	1.79	Poor Performance (PP)
Probabilistic Reasoning	1.76	Poor Performance (PP)
Reasoning about Expressions	2.79	Fair Performance (FP)
Reasoning about Equations and Inequalities	2.63	Fair Performance (FP)
Geometric and Measurement Reasoning	2.29	Fair Performance (FP)
Reasoning about Population Samples and Comparing Populations	1.45	Poor Performance (PP)
Overall Mean	**2.16**	**Fair Performance (FP)**

Legend:

4.00-5.00 points	Outstanding Performance (OP)	1.00-1.99 points	Poor Performance (PP)
3.00-3.99 points	Satisfactory Performance (SP)	0.00-0.99 points	Very Poor Performance (VPP)
2.00-2.99 points	Fair Performance (FP)		

3.2 The Error categories of students in solving word problems before the implementation of the PACE technique

3.2.1 Proportional relationships

Result on proportional relationships shows that 20 (52.63%) errors in proportional relationships were along transformation, 10 (26.32%) were along processing, 5 (13.16%) were along encoding, 2 (5.26%) were along reading, 1 (2.63%) was along comprehension and none of the students had no error in solving this question.

Furthermore, the errors imply that 20 (52.63%) or more than half of the students committed transformation errors. This means that the students were not able to start the calculation correctly. The students failed to state the working equation or expression to start their calculation. These students unsuccessfully transform their ideas into mathematical expressions which eventually trapped them from proceeding to the next stage of the computation. This further means that the students tend to manifest the insufficient know-how skill in dealing with the given problem. The errors encountered were most likely due to poor mastery of the expected competencies.

Table 2. Error Categories in Proportional Relationships Before the Implementation of the PACE Technique

ERROR CATEGOIES BEFORE THE IMPLEMENTATION OF PACE						
Error Categories in Proportional Relationship						
	Reading Error	Comprehension Error	Transformation Error	Processing Error	Encoding Error	No Error
No. of Errors	2	1	20	10	5	0
Rate	5.26%	2.63%	52.63%	26.32%	13.16%	0.00%
Error Categories in Reasoning with Rational Numbers						
No. of Errors	4	13	10	9	2	0
Rate	10.53%	34.21%	26.32%	23.68%	5.26%	0.00%
Error Categories Probabilistic Reasoning						
No. of Errors	1	20	7	7	3	0
Rate	2.63%	52.63%	18.42%	18.42%	7.89%	0.00%
Error Categories in Reasoning about Expression						
No. of Errors	0	5	11	13	7	2
Rate	0.00%	13.16%	28.95%	34.21%	18.42%	5.26%
Error Categories in Reasoning about Equations and Inequalities						
No. of Errors	3	5	11	9	8	2
Rate	7.89%	13.16%	28.95%	23.68%	21.05%	5.26%
Error Categories in Geometric and Measurement Reasoning						
No. of Errors	1	7	16	9	4	1
Rate	2.63%	18.42%	42.11%	23.68%	10.53%	2.63%
Error Categories in Reasoning about Population Samples and Comparing Populations						
No. of Errors	2	19	16	1	0	0
Rate	5.26%	50.00%	42.11%	2.63%	0.00%	0.00%

Based on the actual calculations of the students, 20 (52.63%) of them had transformation errors and were unable to divide $\frac{1}{8}$ by $\frac{1}{4}$. These students failed to use the correct operation which is supposed to be division and they interchanged the numerator and the denominator. This was probably because of being careless and having lack of mastery on the topic.

Moreover, 10 (26.32%) students almost completed the calculation but they incurred processing errors specifically in changing the division sign to multiplication sign and in rewriting the other fraction to its reciprocal. In the end, they failed to simplify and obtain the expected answer. This was possibly due to carelessness and poor recall of “keep-change-flip” or “multiplying by the reciprocal”. Furthermore, there were 5 (13.16%) students who almost got a perfect score on this problem. These students encountered encoding errors as they left their final calculations as they were without stating that “Wanda has read $\frac{1}{2}$ of her book in 1 hour”. This implies that the students failed to state their answer following the correct format and units. This was most likely brought by carelessness and lack of obedience on the “rule of thumb” when it comes to solving word problems. Likewise, only three 3 (7.89%) students had reading and comprehension errors. These students were not able to read the problem correctly and failed to identify what is given and asked in the problem and relate the problem to a specific topic which can help them on how to proceed with the calculations. This was probably caused by laziness and lack of mathematical understanding. And lastly, it is saddening to mention that none of the 38 students obtained no errors or full marks on this problem which only implies that none of them have fully mastered this topic. These findings about the error categories of students in proportional relationships tend to suggest that the students failed to acquire the needed competencies and skills in solving proportions.

The findings of the study contradict Hariyani (2018) who concluded that students could solve arithmetic problems which include ratios and proportions. He further explained that his respondents did not encounter errors in reading, comprehension, transformation and processing but encountered encoding errors. His findings contribute in the importance of encoding stage in word problem solving.

3.2.2. Reasoning with rational numbers

It shows that 13 (34.21%) errors in reasoning with rational numbers were along comprehension, 10 (26.32%) were along transformation, 9 (23.68%) were along processing, 4 (10.53%) were along reading, 2 (5.26%) were encoding and none of them had no errors in dealing with the given problem. It can be gleaned from the table that majority of the students encountered errors along comprehension (13 or 34.21%), transformation (10 or 26,32%) and processing (9 or 23.68%); comprehension being the highest. It only means to say that students most likely to have poor mastery in reasoning with rational numbers as they were unsuccessful in relating the problem to a specific topic to find directions as to how they are going to target the problem. Because they found it difficult to recall the topic, they failed to write the starting equation or expression and eventually stuck them from showing their complete calculations. The errors were probably due to lack of higher order thinking skills.

Considering the calculations, the students presented on their paper and the errors presented on the table, 13 (34.21%) of them had comprehension errors. These students were confused as to which topic they are going to use so that they can solve the given problem; some related this topic to basic operations which was too general and many of them stopped right after writing what was given and asked from the problem. This was most likely brought by poor recall.

In addition, 10 (26.32%) students encountered transformation errors. These students were not able to multiply $\frac{3}{4}$ by 6 and $\frac{3}{2}$ by 5 correctly. Many of them did not show any working equation or expression. These results tend to suggest that the students lack mathematical skills specifically multiplying fractions. This was also most likely due to lack of critical ability to extract major concepts from a given problem. Furthermore, 9 (23.68%) students had processing errors specifically in adding 4.5°C, 5°C and 17.5°C. This means that the students failed to properly execute the steps in solving the problem. Most of the students were rattled with the timings which stopped them from continuing and finalizing their final calculation. This was most probably because of their confusions and lack of critical thinking. Moreover, only 2 (5.26%) students were successful in completing and showing their complete calculations but unable to write their final answer which is supposed to be “The temperature at 5:00 pm is 17°C”. These students had encoding errors. This means that the students failed to write the answer in an acceptable form including units. This tends to suggest that it was brought by carelessness and excitement in proceeding to the next question. Likewise, there were 4 (10.53%) students who had reading errors as they were not able to identify what was given and asked in the problem. These students were most likely not comfortable seeing fractions in different forms as well as units of measurements combined together. This was probably due to laziness and lack of interest. And lastly, it is sad to note again that none of the 38 students successfully completed and solved this problem. It tends to suggest that none of them had mastered this topic, able to understand mathematical explanations and translate them to useful data.

The findings correlate with the study of Yugdu (2020) when he explicated that most students had difficulty dealing with fractions or rational numbers. He further explained that most students who find it difficult often neglect solving this type of problems. This leads the students not to successfully solve problems related to rational numbers. The findings of the study also support the claim of Abdullah (2015) when he explicated that problem solving is an activity that can generate higher order thinking skills (HOTS) among students however, only some are capable and many have difficulties in fractions. He added that students frequently make errors in encoding (27.58%), followed by processing (27.33%) and transformation (24.17%) when dealing with word problems related to fractions or rational numbers.

3.2.3. Probabilistic reasoning

It can be gleaned from the table that 20 (52.63%) of the errors were along comprehension, 7 (18.42%) were along transformation, 7 (18.42%) were along processing, 3 (7.89%) were along encoding, 1 (2.63%) was along reading and 0 (0.00%) had no errors in solving the given problem.

This means that 20 (52.63%) or majority of the students committed comprehension errors in dealing with probabilistic reasoning. This result tends to suggest that the students were not able to identify and relate the problem to probability which hindered them from starting their calculation and writing the working equation or expression. They had not understood clearly and comprehensively what the problem tried to imply. This was most likely rooted from the lack of understanding of the students with some terms related to probability. In addition, 7 (18.42%) of the students had transformation errors. These students failed to write down the correct working equation or expression of the problem. They were not able to analyze on the problem that among the 20 students, 8 of them are females and 12 are males and out of these 8 female students, 3 are wearing blue shirts and 5 are not. The errors were probably due to lack of understanding and critical thinking.

Moreover, there were also 7 (18.42%) students who incurred processing errors. Instead of using the information that there are 5 female students that are not wearing blue shirts, they mistakenly used 15 and other numerical information such as 3 and 8. This opts to suggest that they had mixed-up the data when they tried to complete the calculation. This was most likely because of carelessness and huge confusion about the given data. Further, 3 (7.89%) of the students almost completed solving the problem but were unable to write their final answer which should be “The probability that a randomly chosen student will be a female not wearing a blue shirt is $\frac{1}{4}$ or 0.25 or 25%”. These students just left their calculations as they were. These errors were possibly brought by carelessness and deficient skill in stating the final answer. Likewise, there was only 1(2.63%) student who met a reading error. This student just left the item unanswered. This implies that this student failed to write any data from the problem. It also tends to suggest that this student was not comfortable to deal with the problem. This was probably due to poor mastery of content. And finally, it is quite alarming as none of the 38 students was able to solve the problem completely. This only tends to suggest that the students haven’t acquired yet the needed competencies and skills on this topic.

The findings of the study conform with Triliana and Asih (2018) when they revealed that students often encountered errors in stages of reading, comprehension and processing when dealing with word problems related to probability. Students made errors in choosing the formula to solve the tasks, understanding what the tasks asked and determining the events. They further recommended that teachers should incorporate error analysis in their lesson designs as it will help in making instructional interventions based on students’ needs.

3.2.4. Reasoning about expressions

It shows that the students incurred 13 (34.21%) processing errors, 11 (28.95%) transformation errors, 7 (18.42%) encoding errors, 5 (13.16%) comprehension errors, 2 (5.26%) no errors and 0 (0.00%) reading error. Based on the errors above, 13 (34.21%) or fewer than half of the students committed processing errors. This implies that the students were able to write the correct working expression; however, failed to correctly write the solution. These students failed to show their correct calculations due to the fact that many of them were careless in executing the correct steps. Instead of writing 16p + 14p + 6p, they just wrote 16 + 14 + 6 which eventually led to 36 which was an incomplete answer; it should be 36p. Also, 11 (28.95%) of the students had transformation errors. These students were unable to write the correct expression which should be (p x 16) + (2p x 7) + (3p x 2). They were confused as what the variable p tried to tell. These students just ignored p and wrote 16 + (2 x 7) + (3 x 2). In addition, 7 (18.42%) students encountered encoding errors. The students attempted to complete their calculations but failed to write the final answer as “The expression that represents Doug’s total points is 36p”. These students just left their final calculations as they were. The errors were most likely due to carelessness and lack of skill in writing the final answer following the expected format.

Moreover, there were 5 (13.16%) students who had comprehension errors. These students failed to connect the question to their background knowledge and unable to start their initial calculations. It opts to suggest that these students easily forgot what they have learned about reasoning about expressions. The errors were probably due to lack of recall and understanding. It is also good to note that there were 2 (5.26%) students who did not encounter any mistake while solving the problem. This result suggests that they had fully mastered and understood reasoning about expressions. And lastly, it is also worth to note that none of the students had reading errors on this problem. It tends to suggest that the students were capable of reading the problem and identifying what was given and asked from the problem. The findings of the study are in conformity with Hall (2007) when he elucidated that the deletion and cancellation errors were very common among the respondents of his study specifically in working on expressions. He further explained that “overgeneralizing” was the main cause of this type of error.

3.2.5. Reasoning about equations and inequalities

It can be seen from the table that 11 (28.95%) errors were along transformation, 9 (26.38%) were along processing, 8 (21.05%) were along encoding, 5 (13.16%) were along comprehension, 3 (7.89%) were along reading and 2 (5.26%) did not encounter any error in targeting the problem. The errors above indicate that 11 (28.95%) of the students had transformation errors. This means that the students unsuccessfully represented x or any variable as the number of children’s books bought and also failed to write the working equation as 2 (1.75) + 0.75x = 5.75. Some of them used a variable but were not able to use it to write the working equation. This was probably caused by carelessness and lack of critical thinking. In addition, 9 (26.38%) students encountered processing errors. This means that these students wrote the correct working equation but unsuccessful in solving for x. Some of their common mistakes were specifically on subtracting both sides by 3.5 and in dividing 2.25 by 0.75. There are a few of these students who confidently solve the equations without the use of the calculator but unable to arrive at the correct answer which is x = 3. Furthermore, there were 8 (21.05%) students who successfully completed and showed their calculations but left them as they were leading them to have encoding errors. This implies that the students failed to write the final answer in an acceptable form. These students most likely forgot to write the final answer as “Carlos bought 3 children’s books” and this was brought by carelessness. Also, 5 (13.16%) students had comprehension errors. This tries to say that these students failed to correlate this problem with something that they know especially with equations. They left their answers empty right after writing what was given and asked in the problem. This was most likely due to deficient mathematical understanding on the given problem and the topic it is related to.

Moreover, there were 3 (7.89%) students who encountered reading errors. These students failed to read the problem correctly and identify what was given and asked from the problem. This also suggests that they were not at ease in solving problems related to equations and inequalities as they had difficulties in understanding some math terminologies related to equations and inequalities. This also means to say that the students did not know what to do with the problem as shown on their empty paper. This was probably because of laziness and lack of interest. And lastly, 2 (5.26%) students successfully completed the calculations and wrote their answers following the acceptable format. This tends to suggest that these students had gained the needed competencies and skills in solving problems related to equations and inequalities. However, in general, the students had shown poor mastery and lack of critical thinking.

The findings of the study are aligned to the study of Clement (2002) when he explained that the most incurred error in solving problems related to equations is transformation. He emphasized that his respondents had difficulty in translating words to algebraic equations. Results also conform with the study of Egodawette (2011) when he found out that in linear equations, most of the students got the correct answer; however, some encountered transformation and processing errors. This means that the students failed to form the correct equation. These also run parallel with the study of Allen (2007) when he revealed that students had trouble solving equations. He also explained that students need to learn and acquire the needed skills and fundamental principles on equations. This would probably troubleshoot the issue.

3.2.6. Geometric and measurement reasoning

It reveals that 16 (42.11%) of the errors in geometric and measurement reasoning were along transformation, 9 (23.68%) were along processing, 7 (18.42%) were along comprehension, 4 (10.53%) were along encoding, 1 (2.63%) was along reading and 1 (2.63%) did not encounter any error in solving the problem. The errors on the table tell that 16 (42.11%) of the students committed transformation errors. This means that fewer than half of the students were able to understand what the question wanted them to find out; but failed to identify and use the needed formula and formulate the working equation to solve the problem. These students did not properly use the formula for the volume of a rectangular prism which is V = l x w x h. Many of these students did not figure out that the swimming pools can be correlated to a rectangular prism which is a 3-dimensional figure. The errors were most likely due to lack of visualization and other critical thinking skills. In addition, 9 (23.68%) students incurred processing errors. This implies that the students were not able to show the complete calculations; they stopped when they already found out the volumes of the swimming pools without getting their difference. There are also some of these students who had mistakenly multiplied 16, 24 and 3 and 16, 24 and 5. These results purport to suggest that the errors were due to lack of analysis on the problem.

Moreover, 7 (18.42%) of the students had comprehension errors. This means that these students were able to read the problem and extract what was given and asked from the problem but were not able to use their prior knowledge specifically on volume of 3D shapes. That is why they failed to recall the formula. The errors incurred were possibly due to poor recall of the topic. Further, 4 (10.53%) students committed encoding errors. This implies that the students failed to write the final answer in the most acceptable form. The students were unable to state that “The 2nd swimming pool is 768 ft^3 more than the 1st swimming pool”. There were also some students who did not care about writing the unit of volume which is supposed to be $ft^{3.}$ These results tend to suggest that it was due to carelessness and excitement in dealing with the last problem. It can also be pointed out that there was only 1 (2.63%) student who had a reading error. This means that this student had poor understanding regarding the problem given, which led him not to write anything on his paper. It can also be said that this student did not know what to do with the given problem. This was most likely caused by lack of understanding on volumes of 3D figures. And lastly, there was only 1 (2.63%) student who did not face any mistake in solving the problem. The results opt to suggest that the students had deficient mastery of the subject matter.

The findings of the study corroborate with Zamzam and Patricia (2018) when he explained that there are so many mistakes students encounter when solving word problems especially in Geometry. His study purports to describe students’ error in problem solving and he was able to reveal that students frequently encountered mistakes in the transformation stage.

3.2.7 Reasoning about population samples and comparing populations

Table 2 presents the error categories of students in reasoning about population samples and comparing populations before the PACE technique had been used. It can be gleaned from the table that 19

(50.00%) of the students had comprehension errors, 16 (42.11%) transformation errors, 2 (5.26%) reading errors and 0 (0.00%) encoding errors and no errors.

From the errors above, it can be said that 19 (50.00%) or exactly half of the students had comprehension errors. This tries to say that majority of the students were not able to relate it to the topic Mean Absolute Deviation. Many of these students stated that this problem was related to mean. This was most likely due to their confusion about mean and mean absolute deviation. In addition, 16 (42.11%) or fewer than half of the students encountered transformation errors. This means that the students failed to start their calculation correctly. These students did not compute for the means. Instead, they played around the values and showed irrelevant calculations on the means. This tends to suggest that the students lack the needed computational skills on finding the means of ungrouped data. Moreover, there were 2 (5.26%) students who failed to read the problem and get the needed information to comprehend the problem. These students just left their papers empty without writing anything. This purports to suggest that the students lack the confidence to at least try reading and solving the given problem. This also suggests that they did not know how to answer the problem at all. Furthermore, there was only 1 (2.63%) student who successfully started the calculation but failed to complete it. This student just stopped when he had shown the means and left the latter calculations empty. It suggests that the student did not know how to proceed with the calculation specifically in getting the differences of the means and the individual raw scores. And lastly, it is saddening to note that none of the students completed the calculations correctly and write the final answer. It opts to suggest that the students did not completely master the lesson. It also suggests, somehow that the problem required a long calculation and the students were not prompt enough that is why they got lost around.

The findings correlate with the study of Saleh et al (2017) when he explained that students are required to have the ability to associate the problems encountered by the previous problem because mathematical concepts are connected. This associates with the lack of mastery of the students in measures of center which caused them not to successfully solve the given word problem related to measures of variability.

3.3 Summary of the error categories of students in solving word problems before the implementation of the PACE technique

Table 3 shows the summary of the error categories of students in solving word problems before the implementation of the PACE Technique. It unveils that 13 (34.21%) errors were along transformation, 10 (26.3%) were along comprehension, 8.29 (21.8%) were along processing, 4.14 (10.9%) were along encoding and 1.86 (4.90%) were along reading. This means that fewer than half of the students encountered transformation errors. This tends to suggest that the students failed to formulate the working equation or remember the formula to start solving the given problem. More so, it can also be deduced from the table that 0.71 (1.89%) completed and correctly solved the given problems. This result opts to suggest that majority of the students committed various errors in the given word problems.

The findings of the study corroborate with Ragma (2014) when he explained that majority of his respondents encountered transformation errors when dealing with word problems in Mathematics. He further said that transformation errors are caused by the insufficient skills of students in reading and comprehending the given problem to successfully formulate the working equations. Results likewise in conformity with the study of Hall (2007) when he elucidated that many students while they understand mathematical concepts are inconsistent at computing mainly because they misread signs or carry out numbers incorrectly or may not write numerals in the correct column. He also added that students have difficulty in transferring knowledge which hinders them to work out the working equation or expression which leads to the correct calculation and final answer.

Table 3. Summary of the error categories of students in solving word problems before the implementation of the PACE technique

Standards/Topics	Error Categories					
	R	**C**	**T**	**P**	**E**	**N**
Proportional Relationships	2	1	20	10	5	0
Reasoning with Rational Numbers	4	13	10	9	2	0
Probabilistic Reasoning	1	20	7	7	3	0
Reasoning about Expressions	0	5	11	13	7	2
Reasoning about Equations and Inequalities	3	5	11	9	8	2
Geometric and Measurement Reasoning	1	7	16	9	4	1
Reasoning about Population Samples and Comparing Populations	2	19	16	1	0	0
Average	1.86	10	13	8.29	4.14	0.71
Rate	4.90%	26.3%	34.21%	21.8%	10.9%	1.89%
Rank	5	2	1	3	4	6

Legend:
R- Reading Error C- Comprehension Error T- Transformation Error
P- Processing Error E- Encoding Error N- No Error

3.4. The level of performance of students in solving word problems after the implementation of the PACE technique

Table 4 reflects the level of performance of students in solving word problems after the implementation of the PACE technique. It can be pointed out from the table that the students had an overall mean score of 4.04 which is interpreted as *outstanding performance.* This result tends to suggest that the students achieved to the optimum the needed skills and competencies in the subject. This can be attributed to the fact that even though all the items were word problems that really require higher-order thinking and mathematical skills, they managed to understand and solve them correctly.

Furthermore, the students still scored the highest in reasoning about expressions with a mean score of 4.45, interpreted as *outstanding performance.* This suggests that the students tend to perform very well in this topic. In contrary, they scored the lowest in reasoning about population samples and comparing populations with a mean score of 3.11 which means *satisfactory performance.* Although they got the lowest score in this topic, the students still managed to work well in solving the given word problem. This still indicates that they had obtained the expected competency in this standard. Since the problem requires series of steps and mastery of both measures of center and variability, the satisfactory performance of the students can be attributed to insufficient time and difficulty of the topic.

Table 4. Level of performance of students in solving word problems after the implementation of the PACE technique

Standards/Topics	Median Score	Descriptive Equivalent
Proportional Relationships	4.32	Outstanding Performance (OP)
Reasoning with Rational Numbers	4.00	Outstanding Performance (OP)
Probabilistic Reasoning	4.00	Outstanding Performance (OP)
Reasoning about Expressions	4.45	Outstanding Performance (OP)
Reasoning about Equations and Inequalities	4.37	Outstanding Performance (OP)

Geometric and Measurement Reasoning	4.00	Outstanding Performance (OP)
Reasoning about Population Samples and Comparing Populations	3.11	Satisfactory Performance (SP)
Overall Median	**4.04**	**Outstanding Performance (OP)**

Legend:

4.00-5.00 points	Outstanding Performance (OP)	1.00-1.99 points	Poor Performance (PP)
3.00-3.99 points	Satisfactory Performance (SP)	0.00-0.99 points	Very Poor Performance (VPP)
2.00-2.99 points	Fair Performance (FP)		

The findings of the study are no longer harmonizing with the study of Elis (2013) stating that the students had moderate performance in algebraic expressions revealing that the students had high competence in pre-algebra, which included algebraic expressions. And lastly, the findings are no longer aligned to the results of the study of Kwon (2017) when he explained that when students are prompted with knowledge or computation questions, students' success rate is 86 percent or even higher; but, when students are prompted with word problems, their success rate dips down to as low as 39 percent. Students find it difficult to deal with word problems however, when they are given guidance and help and the right intervention, they tend to develop and apply the needed higher-order-thinking skills to do the correct calculations and arrive at the correct answer.

3.5 The error categories of students in solving word problems after the implementation of PACE technique

3.5.1. Proportional relationships

Table 5 presents the errors of students along proportional relationships after the implementation of the PACE technique. It shows that 18 (47.37%) of the students did not encounter any mistake in solving the problem, 15 (39.47%) committed encoding errors, 4 (10.53%) incurred processing errors, 1 (2.63%) encountered a transformation error and 0 (0.00%) had reading and comprehension errors. It is worth to note that 18 (47.37%) students obtained no errors or full marks on this problem which tends to suggest that they have already fully mastered this topic. This also suggests that the students had successfully acquired the needed competencies and skills in solving proportions. Also, there were 15 (39.47%) students who almost got a perfect score on this problem. These students encountered encoding errors as they still left their final calculations as they were without stating that "Wanda has read $\frac{1}{2}$ of her book in 1 hour".

Table 5. Error categories in proportional relationships after the implementation of the PACE Technique

ERROR CATEGORIES AFTER THE IMPLEMENTATION OF PACE						
Error Categories in Proportional Relationship						
	Reading Error	**Comprehension Error**	**Transformation Error**	**Processing Error**	**Encoding Error**	**No Error**
No. of Errors	0	0	1	4	15	18
Rate	0.00%	0.00%	2.63%	10.53%	39.47%	47.37%
Error Categories in Reasoning with Rational Numbers						
No. of Errors	0	0	3	8	13	14
Rate	0.00%	0.00%	7.89%	21.05%	34.21%	36.84%
Error Categories Probabilistic Reasoning						
No. of	0	0	1	10	15	12

Errors						
Rate	0.00%	0.00%	2.63%	26.32%	39.47%	31.58%
Error Categories in Reasoning about Expression						
No. of Errors	0	0	1	3	12	22
Rate	0.00%	0.00%	2.63%	7.89%	31.58%	57.89%
Error Categories in Reasoning about Equations and Inequalities						
No. of Errors	0	0	1	4	13	20
Rate	0.00%	0.00%	2.63%	10.53%	34.21%	52.63%
Error Categories in Geometric and Measurement Reasoning						
No. of Errors	0	0	0	11	18	9
Rate	0.00%	0.00%	0.00%	28.95%	47.37%	23.68%
Error Categories in Reasoning about Population Samples and Comparing Populations						
No. of Errors	0	0	11	17	7	3
Rate	0.00%	0.00%	28.95%	44.74%	18.42%	7.89%

This implies that the students failed to state their answer following the correct format and units. This was most likely brought by carelessness and lack of obedience on the "rule of thumb" when it comes to solving word problems. Moreover, 4 (10.53%) students almost completed the calculation but they incurred processing errors specifically in changing the division sign to multiplication sign and in rewriting the other fraction to its reciprocal. In the end, they failed to simplify and obtain the expected answer. This was probably due to carelessness and poor recall of "keep-change-flip" or "multiplying by the reciprocal". This time, only 1 (2.63%) student had a transformation error and unable to divide $\frac{1}{8}$ by $\frac{1}{4}$. This student interchanged the numerator and the denominator. This was possibly because of being careless and having lack of mastery on the topic. And lastly, none of the students encountered reading and comprehension errors. The students, in general, were able to read the problem correctly and identify what is given and asked in the problem and relate the problem to a specific topic which helped them on how to proceed with the calculations.

The findings of the study agree with Hariyani (2018). He found out that students could solve arithmetic problems which include ratios and proportions. He further explained that his respondents did not encounter errors in reading, comprehension, transformation and processing but encountered encoding errors. In this study, fewer than half of the students encountered encoding errors which fully supports the findings of Hariyani which contribute in the importance of encoding stage in word problem solving.

3.5.2. Reasoning with rational numbers

Table 5 reflects the error categories of the students on reasoning with rational numbers after the PACE technique had been implemented. It shows that 14 (36.84%) had no errors in reasoning with rational numbers, 13 (34.21%) were along encoding, 8 (21.05%) were along processing, 3 (7.89%) were along transformation and 0 (0.00%) was along reading and comprehension. It can be gleaned from the table that 14 (36.84%) students successfully completed and solved this problem and wrote the final answer based on the format. This opts to suggest that these students have fully mastered this topic. This also suggests that the students were able to understand mathematical explanations and translate these to useful data. Moreover, 13 (34.21%) students were successful in completing and showing their complete calculations but unable to write their final answer which is supposed to be "The temperature at 5:00 pm is 17°C". These students had encoding errors. This means that the students failed to write the answer in an acceptable form including units. This was most likely brought by carelessness and excitement in proceeding to the next question.

Furthermore, 8 (21.05%) students had processing errors. Many students still mistakenly added 4.5°C, 5°C and 17.5°C. This means that the students still failed to properly execute the steps in solving the problem. They were still confused with the timings which stopped them from continuing and finalizing their final calculation. The errors were probably because of their confusions and lack of critical thinking. In addition, 3 (7.89%) students encountered transformation errors. These students were not able to multiply $\frac{3}{4}$ by 6 and $\frac{3}{2}$ by 5 correctly. One of these 3 students did not show any working equation or expression. This tends to suggest that the students still lack mathematical skills specifically multiplying fractions. This was probably due to lack of critical ability to extract major concepts from a given problem. And lastly, it is good to note that none of the students committed reading and comprehension errors. These students were able to identify what is given and asked in the problem. These students were probably no longer confused as to which topic they are going to use so that they can solve the given problem.

The findings are now contradicting the findings of Hall (2007) when he explicated that most students had difficulty dealing with fractions or rational numbers. He further explained that most students who find it difficult often neglect solving this type of problems. This leads the students not to successfully solve problems related to rational numbers. In this study, fewer than half of the students successfully dealt with the given word problem which tends to suggest that they did not have difficulties with fractions or rational numbers.

Conversely, the results disagree with the claim of Abdullah (2015) when he explicated that problem solving is an activity that can generate higher order thinking skills (HOTS) among students however, only some are capable and many have difficulties in fractions. He added that students frequently make errors in encoding (27.58%), followed by processing (27.33%) and transformation (24.17%) when dealing with word problems related to fractions or rational numbers. In connection to this study, since 36.84 percent of the students did not encounter any mistake in solving the given problem, it opts to suggest that many can students are now capable of dealing with word problems related to fractions and rational numbers.

3.5.3. Probabilistic reasoning

In probabilistic reasoning, it can be gleaned from the table that 15 (39.47%) of the students had encoding errors, 12 (31.58%) had no error, 10 (26.32%) had processing errors, 1 (2.63%) had transformation error and 0 (0.00%) had reading and comprehension errors. In the table, 15 (39.47%) students almost completed solving the problem but were unable to write their final answer which should be "The probability that a randomly chosen student will be a female not wearing a blue shirt is $\frac{1}{4}$ or 0.25 or 25%". These students just left their calculations as they were. These errors were possibly brought by carelessness and deficient skill in stating the final answer. Furthermore, 12 (31.58%) students were able to solve the problem completely and they had written their final answer following the standard format. It purports to suggest that the students have already acquired the needed competencies and skills on this topic. Moreover, there were also 10 (26.32%) students who incurred processing errors. These students still misused the numerical data given in the problem. It opts to suggest that they had mixed-up the data when they tried to complete the calculation. This was most likely because of carelessness and huge confusion about the given data.

This time, only 1 (2.63%) student had a transformation error. The student failed to write down the correct working equation or expression of the problem. The student was not able to analyse on the problem that among the 20 students, 8 of them are females and 12 are males and out of these 8 female students, 3 are wearing blue shirts and 5 are not. The error was probably due to lack of understanding and critical thinking. And lastly, it is good to note that none of the students encountered reading and comprehension errors. The students made sure that they had written what is given and asked in the problem as well as the topic that the problem is connected to. They had understood clearly and comprehensively what the problem tried to imply. This suggests that the students were, somehow, comfortable to deal with the problem. This was possibly due to a good mastery of content.

The findings of the study do not run parallel to the study of Triliana and Asih (2018) when they revealed that students often encountered errors in stages of reading, comprehension and processing when dealing with word problems related to probability. Students made errors in choosing the formula to solve the tasks, understanding what the tasks asked and determining the events. They further recommended that teachers should incorporate error analysis in their lesson designs as it will help in making instructional interventions based on students' needs. In this study, 39.47 percent of the students had encoding errors and 31.58 percent of them did not have errors in solving the given word problem. This tends to suggest that majority of the students did not have difficulties and did not have reading, comprehension and processing errors in dealing with word problems related to probability.

3.5.4. Reasoning about expressions

The reasoning about expression shows that the 22 (57.89%) students did not incur any mistake in solving the given problem, 12 (31.58%) incurred encoding errors, 3 (7.89%) committed processing errors, 1 (2.63%) encountered a transformation error, and 0 (0.00%) had reading and comprehension errors. It can be deduced from the table that 22 (57.89%) or majority of the students did not encounter any mistake while solving the problem. It tends to suggest that they had fully mastered and understood reasoning about expressions. It also suggests that these students were very at ease when solving the given problem as they were able to write what is given and unknown in the problem, they also correctly identified the topic which the problem is connected to; they had also successfully used the data in the problem to write the working expression and simplify it to get the final answer and lastly, they were able to write their answer in the most acceptable form.

In addition, 12 (31.58%) students encountered encoding errors. The students attempted to complete their calculations but failed to write the final answer as "The expression that represents Doug's total points is 36p". These students still left their final calculations as they were. There are some of them who wrote their final answer as "The expression that represents Doug's total points is 36" and it was seen on their calculation that they had written 36p. The errors were most likely due to carelessness and lack of skill in writing the final answer following the expected format. Moreover, 3 (7.89%) of the students committed processing errors. This tends to suggest that the students were able to write the correct working expression; however, failed to correctly write the solution. These students failed to show their correct calculations due to the fact that many of them were careless in executing the correct steps. Instead of writing 16p + 14p + 6p, they just wrote 16 + 14 + 6 which eventually led to 36 which was an incomplete answer; it should be 36p. Also, 1 (2.63%) student had a transformation error. The student was unable to write the correct expression which should be (p x 16) + (2p x 7) + (3p x 2). The student was probably confused as what the variable p tried to tell.

And lastly, none of the students encountered reading and comprehension errors. This opts to suggest that the students were capable of reading the problem and identifying what is given and asked in the problem. These students also successfully connected the question to their background knowledge and able to start their initial calculations. This also means to say that these students easily recalled what they have learned about reasoning about expressions. In general, the students having no reading and comprehension errors were most probably due to good recall and understanding.

The findings of the study oppose with Hall (2007) when he elucidated that the deletion and cancellation errors were very common among the respondents of his study specifically in working on expressions. He further explained that "overgeneralizing" was the main cause of this type of error. 57.89 percent or majority of the students were able to successfully solve the given word problem and it tends to suggest that there was no "overgeneralizing" that happened in dealing with the question related to expressions.

3.5.5. Reasoning about equations and inequalities

In reasoning about equations and inequalities after the implementation of the PACE technique, it can be seen from the table that 20 (52.63%) of the students had no error in working on the given problem, 13 (34.21%) had encoding errors, 4 (10.53%) had processing errors, 1 (2.63%) had a transformation error and 0 (0.00%) had reading and comprehension errors. The errors above indicate that 20 (52.63%) students successfully completed the calculations and wrote their answers following the acceptable format. This result tends to suggest that these students had gained the needed competencies and skills in solving problems related to equations and inequalities. The students had probably shown good mastery and critical thinking.

Furthermore, there were 13 (34.21%) students who successfully completed and showed their calculations but left them as they were leading them to have encoding errors. This purports to suggest that the students failed to write the final answer in an acceptable form. These students forgot to write the final answer as "Carlos bought 3 children's books". This was most probably brought by carelessness. In addition, 4 (10.53%) students encountered processing errors. This means that these students wrote the correct working equation but unsuccessful in solving for x. Some of their common mistakes were still specifically on subtracting both sides by 3.5 and in dividing 2.25 by 0.75. This was most likely due to lack of understanding about inverse operations and cancellation. Moreover, 1 (2.63%) of the students had a transformation error. This means that the student unsuccessfully represented x or any variable as the number of children's books bought and also failed to write the working equation as $2 (1.75) + 0.75x = 5.75$. This was probably caused by carelessness and lack of critical thinking.

And lastly, none of the students committed reading and comprehension errors. This result tends to suggest that these students had successfully read and identified what was given and asked in the problem. This also means that they were comfortable in solving problems related to equations and inequalities as they did not have difficulties in understanding some math terminologies related to equations and inequalities. This also suggests that the students knew what to do with the problem as evidenced by what they had written on their paper. It can also be said that students were able to correlate this problem with something that they knew especially with equations. This success was probably due to sufficient mathematical understanding on the given problem and the topic it is related to.

The findings of the study are not aligned to the study of Clement (2002), Egodawette (2011) and Allen (2007) when they revealed that students had trouble and incurred transformation and processing errors in solving problems related to equations. In connection to this study, 52.63 percent or majority of the students had no errors in solving word problems related to equations and inequalities. It tends to suggest that they did not have difficulties or troubles in dealing with questions related to this topic.

3.5.6. Geometric and measurement reasoning

It reveals that 18 (47.37%) of the errors in geometric and measurement reasoning were along encoding, 11 (28.95%) were along transformation, 9 (23.68%) had no errors, and 0 (0.00%) was along reading, comprehension and transformation. The table highlights that 18 (47.37%) students committed encoding errors. This implies that the students failed to write the final answer in the most acceptable form. The students were unable to state that "The 2nd swimming pool is 768 ft^3 more than the 1st swimming pool". There were also some students who did not care about writing the unit of volume which is supposed to be $ft^{3.}$ This was most probably due to carelessness and excitement in dealing with the last problem. In addition, 11 (28.95%) students incurred processing errors. This tends to suggest that the students were not able to show the complete calculations; they stopped when they already found out the volumes of the swimming pools without getting their difference. There are also some of these students who had mistakenly multiplied 16, 24 and 3 and 16, 24 and 5. The errors were most likely due to lack of analysis on the problem. Furthermore, there were 9 (23.68%) students who did not face any mistake in solving the problem. This opts to suggest that these students had sufficient mastery of the subject matter. It also suggests that they had also acquired the needed skills and competencies in dealing with word problems related to geometric and measurement reasoning especially on volumes of rectangular prism. And lastly, it is good to note that none of the students encountered reading, comprehension and transformation errors. This purports to suggest that the students had very good understanding regarding the problem given, which led them to write what is given and unknown. It can also be said that the students knew what to do with the given problem. This was caused by sufficient understanding on volumes of 3D figures. This also means that the students were able to understand what the question wanted them to find out and identify and use the needed formula and formulate the working equation to solve the problem. This time, the students properly used the formula for the volume of a rectangular prism which is V = l x w x h. They were also able to figure out that the swimming pools can be correlated to a rectangular prism which is a 3-dimensional figure. The success was most likely due to good visualization and other critical thinking skills.

The findings of the study do not correlate with Zamzam and Patricia (2018) when he explained that there are so many mistakes students encounter when solving word problems especially in Geometry. His study purports to describe students' error in problem solving and he was able to reveal that students frequently encountered mistakes in the transformation stage. In relation to this study, none of the students encountered transformations errors in solving word problems related to geometric and measurement reasoning. In fact, 47.37 percent of the students had encoding errors and 23.68 percent had no errors. This tends to suggest that many of the students did not encounter so much difficulty in solving problems related to this topic.

3.5.7. Reasoning about population samples and comparing populations

In reasoning about population samples and comparing populations, it can be gleaned from the table that 17 (44.74%) of the students had processing errors, 11 (28.95%) had transformation errors, 7 (18.42%) had encoding errors, 3 (7.89%) had no error and 0 (0.00%) had reading and comprehension errors.

From the errors above, it can be said that 17 (44.74%) of the students had processing errors. These students successfully started the calculation but failed to complete it. These students just stopped when they had shown the means and left the latter calculations empty. It opts to suggest that the student did not know how to proceed with the calculation specifically in getting the differences of the means and the individual raw scores. In addition, 11(28.95%) of the students encountered transformation errors. This means that the students failed to start their calculation correctly. These students did not compute for the means. Instead, they played around the values and showed irrelevant calculations on the means. This tends to show that the students lack the needed computational skills on finding the means of ungrouped data. Moreover, 7 (18.42%) students encountered encoding errors. These students tried to show and do the complete calculations but failed to write the final answer following the standard format. It suggests that the students were careless and rushed on this part of the question and forgot to write the final answer. Likewise, 3 (7.89%) students completed the calculations correctly and wrote the final answer using the expected format. It suggests that these students completely mastered the lesson. It also suggests that although the problem required a long calculation, the students were prompt enough not to get lost in solving the problem. And lastly, it is worth to note that none of the students incurred reading and comprehension errors. These students had successfully and carefully read the problem and deduced the needed information to comprehend the problem. These students wrote what was given and asked in the problem as well as the topic the problem was related to which was specifically Mean Absolute Deviation. They can now differentiate it from mean. This implies that the students had the confidence to at least try reading and solving the given problem. This also means that they knew how to answer the problem.

The findings still corroborate with the study of Saleh et al (2017) when he explained that students are required to have the ability to associate the problems encountered by the previous problem because mathematical concepts are connected. This associates with the lack of mastery of the students in measures of center which caused them not to successfully solve the given word problem related to measures of variability. In connection to this study, majority of the students encountered transformation (28.95%) and processing errors (44.74%). This opts to suggest that the students had issues with measures of center and variability as many of them were unsuccessful in solving for the mean and were unable to complete solving for the mean absolute deviation.

3.6. Summary of the error categories of students in solving word problems after the implementation of the PACE technique

Table 6 shows the summary of the error categories of students in solving word problems after the implementation of the PACE Technique. It unveils that 14 (36.84%) students did not encounter errors in solving the given word problems, 13.29 (34.96%) had encoding errors, 8.14 (21.43%) had processing errors, 2.57 (6.77%) had transformation errors and none of them had reading and comprehension errors. It can be deduced from the table that 36.84 percent or fewer than half of the students encountered no errors. This tends to suggest that the students successfully solved the given word problems with ease and confidence. This also suggests that the students were able to master the topics and met the standards. It is also reflected on the table that none of the students had difficulties reading and comprehending the given word problems. This result opts to suggest that the students had successfully and carefully read the problem and deduced the needed information to comprehend the problem. These students wrote what was given and asked in the problem as well as the topic the problem was related to.

The findings of the study do not agree with the study of Ragma (2014) when he explained that majority of his respondents encountered transformation errors when dealing with word problems in Mathematics. He further said that transformation errors are caused by the insufficient skills of students in reading and comprehending the given problem to successfully formulate the working equations. In connection to this study, 36.84 percent of the students did not have errors in solving the given word problems and 34.96 percent of the students had encoding errors. This obviously counters the claims of Ragma (2014) since transformation was not the most incurred error in this study.

Table 6. Summary of the error categories of students in solving word problems after the implementation of the PACE technique

Standards/Topics	Error Categories					
	R	**C**	**T**	**P**	**E**	**N**
Proportional Relationships	0	0	1	4	15	18
Reasoning with Rational Numbers	0	0	3	8	13	14
Probabilistic Reasoning	0	0	1	10	15	12
Reasoning about Expressions	0	0	1	3	12	22
Reasoning about Equations and Inequalities	0	0	1	4	13	20
Geometric and Measurement Reasoning	0	0	0	11	18	9
Reasoning about Population Samples and Comparing Populations	0	0	11	17	7	3
Average	0	0	2.57	8.14	13.29	14
Rate	0.00%	0.00%	6.77%	21.43%	34.96%	36.84%
Rank	6	6	4	3	2	1

Legend:
R- Reading Error C- Comprehension Error T- Transformation Error
P- Processing Error E- Encoding Error N- No Error

The findings do not also run parallel to the study of Chiphambo and Mtsi (2021) when they elucidated that many students while they understand mathematical concepts are inconsistent at computing mainly because they misread signs or carry out numbers incorrectly or may not write numerals in the correct column. Students have difficulty in transferring knowledge which hinders them to work out the working equation or expression which leads to the correct calculation and final answer. In relation to this study, majority of the students understand mathematical concepts and also consistent with their calculations. The students had no difficulties in transferring knowledge to work out the initial calculation as evidenced by their minimal mistakes in solving word problems in Mathematics.

3.7. Comparison on the level of performance of students in solving word problems before and after the implementation of the PACE technique

Table 7 shows the comparison on the level of performance of students in solving word problems before and after the implementation of the PACE technique. It is presented on the table that the students had a mean of 15.11 before and 28.24 after the implementation of the PACE. It can also be gleaned from the table that the t_{value} is greater than t_{crit} which means that the null hypotheis (H_o) which was "There is no significant difference between the level of performance of students before and after using the PACE technique" was rejected. This tends to suggest that there was a significant difference between the performance of students before and after using the PACE technique. The students had significantly improved and were able to reduce their errors in word problem solving. It can finally be said that the PACE technique was effective in improving the performance of students and in reducing their errors in word problem solving.

Table 7. *Comparison on the level of performance of students in solving word problems before and after the implementation of the PACE technique*

	N	Mean Score	Std. Deviation	t_{value}	t_{crit} $(\alpha = 0.05)$	P Value	Decision
Before	38	15.11	6.24	27.43	2.03	3.46	Since $t_{value} > t_{crit}$, Reject H_o
After	38	28.24	4.81				

The findings of the study run parallel to the study of Sinay (2018) when she explicated that it is important to design effective instructional strategies to present evidence-based mathematics interventions to help address the challenge of students struggling in mathematics. She has discussed many intervention strategies in her study and she finally generalized that with all the interventions, there should be a shared commitment to the intervention program for it to be successful. In relation to this study, the PACE technique served as an instructional strategy to address the issue on the low performance of students in mathematics. The findings suggest that the PACE technique has greatly helped the students to reduce their errors in word problem solving which eventually resulted to the significant improvement of the students in solving word problems.

3.8. Comparison on the number of errors of students in solving word problems before and after the implementation of the PACE technique

Table 8 presents the topics and error categories of students and their respective means, standard deviations and statistical values before and after the implementation of the PACE Technique. It also shows the decision as to reject or accept the null hypothesis which was "There is no significant difference between the number of errors of students in solving word problems along reading, comprehension, transformation, process and encoding before and after using the PACE technique".

It can be deduced from the table that the null hypothesis (H_0) was rejected along reading, comprehension, transformation and encoding errors. This means that there was a significant difference between the number of errors of students in solving word problems along these errors before and after using the PACE technique. This tends to suggest that the students had significantly reduced their reading, comprehension, transformation and encoding errors after the implementation of the PACE technique. It also suggests that the PACE technique was effective in reducing errors in word problem solving specifically in the reading, comprehension, transformation and encoding stages. On the other note, the null hypothesis (H_0) was accepted along processing error. This means that that there was no significant difference between the number of errors of students in solving word problems along processing errors before and after using the PACE technique.

Table 8. Comparison on the number of errors in solving word problems before and after the implementation of the PACE technique

Error Categories	**Mean (Before)**	**Mean (After)**	**Std Dev Before**	**Std Dev After**	**t_{value}**	**t_{crit} ($\alpha = 0.0$**	**P Value**	**Decision**
Reading Error	1.86	0.00	1.35	0	3.65	2.45	0.01	Since $t_{value} > t_{crit}$, Reject H_o
Comprehension Error	10	0	7.42	0	3.57	2.45	0.01	Since $t_{value} > t_{crit}$, Reject H_o
Transformation Error	13	2.57	3.16	3.82	5.25	2.45	0.01	Since $t_{value} > t_{crit}$, Reject H_o
Processing Error	8.29	8.14	3.68	5.01	0.04	2.45	0.01	Since $t_{value} < t_{crit}$, Accept H_o
Encoding Error	4.14	13.29	2.79	3.40	6.85	2.45	0.01	Since $t_{value} > t_{crit}$, Reject H_o

This tends to suggest that the PACE technique was not appropriate and effective in reducing processing errors.

The findings of the study corroborate with Ragma (2014) when he elucidated that the different error categories and their causes can be addressed with the use of an instructional intervention. As related to this study, the PACE technique served as an instructional intervention that helped the students in reducing their errors and in improving their performance in word problem solving.

The Theory of Errors and Error Categories of Newman (1977) explains that it is very common for students to encounter errors when confronted with mathematical problems. These errors can be reading, comprehension, transformation, processing and encoding and they are often brought by their low interest in the subject, high anxiety, negative attitude and mindset, lack of recall, misconception and misinterpretation, poor mastery and carelessness.

It is in this light that Newman (1977) suggested and devised a series of prompts to help teachers determine where students are making mistakes in problem solving. The prompts that were included were “Please read the question to me. If you don’t know a word, leave it out.”, “Tell me what the question is asking you to do.”, “Tell me how you are going to find the answer.”, “Show me what to do to get the answer. Talk aloud as you do it so that I can understand how you are thinking.” and “Now write the answer to your question.” It is very evident that these prompts are in accordance to the five major errors of students when dealing with word problems namely reading, comprehension, transformation, processing and encoding. These prompts are a perfect opportunity for the students to look back and check what the question was asking them to do. This will also solve their problem on encountering silly mistakes of using amazing strategies and reasoning but leaving out a small and vital piece of information right at the end.

The findings of the study conform to the Theory of Errors and Error Categories of Newman (1977). The students had a fair performance in solving word problems before the implementation of the PACE technique. In addition, when they made a mistake in problem solving, it was not necessarily because of flaws in their processing skills but they also made an error in the reading or interpretation of the question, the identification of which strategies to employ, execution of their calculations or in the final communication of their answer. With the use of Newman's Error Analysis Tool, the researcher was able to identify where the respondents went wrong in their problem solving. He used this information to accurately assess where the students need extra assistance and implemented the PACE technique to mainly address the main issues.

The regular use of the prompts gave his students a good framework for solving problems and gave them great opportunities to both understand and use mathematical language more effectively. With this, they have significantly improved and were able to reduce their errors in word problem solving. The effectiveness of the PACE technique in improving the performance and reducing the errors of students in solving word problems was mainly because of the proper execution and application of Newman's constructs and prompts.

4. Conclusions

Based on the results of the study, the following conclusions were drawn. The 7th Grade students possess inadequate skills to competently deal with proportional relationships, reasoning with rational numbers, probabilistic reasoning, reasoning about expressions, reasoning about equations and inequalities, geometric and measurement reasoning and reasoning about population samples and comparing populations before using the PACE technique. It is recommended that the students should exert more effort and spend more time in understanding the different topics in Mathematics by doing drills and exercises. On the other hand, Mathematics teachers should also suit their teaching strategies and techniques based on their students' needs and interests.

In addition, insufficient knowledge and skills in mathematical concepts, inadequate recall as well as lack of mastery made the student unsuccessful in completing the problem-solving processes before the implementation of the PACE technique. To address the issue, the students should be more aware of their errors in dealing with word problems in math and take the initiative to identify and correct them. Teachers should implement strategies on how to successfully deal with word problems in math like error analysis using PACE technique. Furthermore, the 7th Grade students tend to have acquired the needed competencies and skills in proportional relationships, reasoning with rational numbers, probabilistic reasoning, reasoning about expressions, reasoning about equations and inequalities, geometric and measurement reasoning and reasoning about population samples and comparing populations after using the PACE technique. So, the students should continue on mastering all the topics in Mathematics and enhance more their competencies and skills in word problem solving. Also, teachers should supplement and enhance their students' learning so that they will not easily forget what they have learned.

Moreover, the 7th Grade students tend to have developed and gained enough knowledge and skills in mathematical concepts, were able to properly recall mathematical ideas and formulas and were able to master the topics which enabled them to start and complete the problem-solving processes and which led them to successfully finish all the stages of problem solving in proportional relationships, reasoning with rational numbers, probabilistic reasoning, reasoning about expressions, reasoning about equations and inequalities, geometric and measurement reasoning but not so in reasoning about population samples and comparing populations after the implementation of the PACE technique. It is recommended that the students should continue using the PACE technique to reduce their errors in reasoning about population samples and comparing populations. The teachers should also revisit the performance of students in this topic and continue implementing the PACE technique to reduce their errors. Implement other error analysis techniques if there is a need.

Likewise, the PACE technique helps students to develop and gain knowledge and skills in mathematical concepts, recall important ideas and formulas and master the topics which can eventually result to higher performance and reduced errors of students in word problem solving. So, Teachers should do error analysis and implement the PACE technique to reduce or eliminate the errors of students in word problem solving. It is most likely to enhance student understanding and application in the reading, comprehension, transformation and encoding stages but not in the processing stage. With this, teachers should emphasize the different stages of word problem solving namely reading, comprehension, transformation, processing and encoding when teaching Mathematics and make the students get used to it.

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