QUBAHAN ACADEMIC JOURNAL
VOL. 6, NO. 4, November 2026
https://doi.org/10.48161/qaj.v6n4a2685

Visual Imagery as a Cognitive Gateway: Students' Mathematical Problem-Solving Process in Geometry Assignments Based on Local Culture

Nanang Nabhar Fakhri Auliya [1], Scolastika Mariani [1*], Bambang Eko Susilo [1], Yohanes Leonardus Sukestiyarno [1] and Emi Pujiastuti [1]

[1] Department of Mathematics Education, Faculty of Mathematics and Natural Sciences, Universitas Negeri Semarang, Semarang 50229, Indonesia.

* **Corresponding author:** mariani.mat@mail.unnes.ac.id.

ABSTRACT: This qualitative descriptive study examined how three eighth-grade students from different visual imagery questionnaire categories coordinated diagrams, calculations, and explanations when solving a locally contextualized roof-related geometry problem based on the Pythagorean theorem. All 31 students in one class at MTs Manbaul Huda Grobogan completed a 14-item visual imagery questionnaire. One student each from the high, moderate, and low categories was then purposively selected, with the teacher's recommendation based on communication ability. Data comprised the students' written solutions and retrospective semi-structured interviews. The high-visual-imagery student used the height and half-base in an appropriate right-triangle model, rechecked the result, judged its reasonableness, and applied the model to an increased base. The moderate-visual-imagery student initially used the full 8-metre base, then corrected it to 4 metres, attributing the correction to reviewing the diagram, but could not explain the effect of a longer base. The low-visual-imagery student wrote the correct final answer of 5 metres through an invalid setup and arithmetic errors, and reported not checking the answer. These cases show that numerical correctness, diagram use, and reported mental imagery can diverge, so none alone is sufficient evidence of imagery processes. The findings describe only these three students on a single, guided task. They do not establish that visual imagery caused the differences, nor do they represent all students in each category.

Keywords: Visual Imagery, Mathematical problem solving, Ethnomathematics, Pythagorean Theorem.

I. INTRODUCTION

Mathematical problem-solving skills are an essential competency in 21st-century mathematics learning, playing a central role in developing critical thinking, reasoning, and the application of concepts in real-life contexts. Modern mathematics education no longer focuses solely on procedural mastery; instead, it places problem-solving at the heart of learning activities, emphasizing students' in-depth thinking processes [1, 2]. Polya [3] describes problem solving through four phases: understanding the problem, devising a plan, carrying out the plan, and looking back. These phases direct attention to how a solver interprets the problem, selects and implements a strategy, and examines the resulting solution. Various studies show that students' mathematical problem-solving abilities are still relatively low, especially in solving non-routine problems that require reasoning and integration of various representations [4, 5]. The OECD's PISA 2022 report shows that 31% of students across OECD countries performed below Level 2 in mathematics [1]. These challenges indicate that difficulties are not limited to conceptual gaps, but extend to students' thinking processes during problem solving, including the role of internal cognitive mechanisms such as Visual Imagery [6].

Theoretically, mathematical problem-solving ability can be assessed not only from the final result, but also from the process that students go through. Several experts have proposed relatively consistent problem-solving

indicators: (1) understanding the problem, (2) planning a strategy, (3) mathematical modeling, (4) implementing the solution, and (5) evaluating or reflecting on the results [7–9]. These indicators reflect that problem solving is a complex process that involves simultaneous cognitive and metacognitive engagement [10]. One cognitive factor that plays a crucial role in this process is visual imagery. Visual imagery is an individual's ability to form, maintain, and manipulate mental images of mathematical objects or concepts. Kosslyn [11] describes mental imagery as representing information in the absence of the corresponding external stimulus. In mathematics education, Presmeg [12] discusses visualization in relation to mental images and external representations used in mathematical activity. These perspectives provide a basis for examining students' descriptions of imagined geometric structures alongside the diagrams they produce, while distinguishing the two forms of representation. Recent research shows that visual imagery helps students construct problem representations, reduces cognitive load, and improves mathematical reasoning; conversely, limitations in constructing accurate mental representations can lead to conceptual, procedural, and representational errors in problem solving [6, 13]. However, visual imagery should be distinguished from drawing ability, geometric knowledge, and general mathematics ability. Mental imagery concerns internal representations, whereas drawings are external representations [14]. Research distinguishing schematic and pictorial representations also indicates that representation use, spatial ability, and mathematical performance should not be treated as interchangeable [15]. Accordingly, this study does not treat a correct diagram or numerical answer alone as sufficient evidence of a particular mental imagery process.

In mathematics education, Visual Imagery indicators include: (1) image formation (building mental images); (2) image maintenance and inspection (maintaining and checking mental images); (3) image transformation (transforming mental images); (4) representational conversion (converting between representations); and (5) using mental images as a reasoning and problem-solving strategy [11, 16–18]. These indicators provide a framework for examining possible imagery processes, although external representations and correct solutions alone are insufficient to establish the internal processes involved. In addition to cognitive factors, the learning context also plays a crucial role in supporting student understanding. Integrating local culture into mathematics learning is an approach that can enhance the meaningfulness of learning [19, 20]. D'Ambrosio's ethnomathematics programme examines mathematical knowledge and practices in relation to the cultural contexts in which they develop [21]. This perspective provides a rationale for considering everyday activities and locally familiar structures as contexts for mathematical problems. Local cultures contain a variety of patterns, shapes, and structures that can serve as contexts for learning, particularly in geometry.

Ruamba et al. [6] reviewed research on visual and multimodal representations in relation to cognitive load and mathematical problem solving, whereas Munawwir et al. [13] examined mental imagery and types of geometric errors in three mathematics education students selected on the basis of their errors. Building on these studies, the present research examines three eighth-grade students selected from different visual imagery questionnaire categories as they solve the same roof-related task. Its specific focus is the correspondence and divergence among written diagrams, calculations, and retrospective explanations, particularly when a student corrects a geometric relationship after reviewing a diagram or obtains a correct numerical answer through an incorrect procedure. The study's contribution is to illustrate how numerical correctness, use of an external diagram, and reported mental imagery can diverge within a solution. Comparing written work with retrospective explanations provides a basis for distinguishing evidence of geometric understanding from evidence relevant to imagery, without attributing mathematical errors solely to visual imagery.

Based on these considerations, this study aims to analyze the thinking processes of three selected students, one from each of the high, moderate, and low Visual Imagery questionnaire categories, when solving a locally contextualized roof-related geometry problem. The analysis examines their written responses and retrospective interview explanations to describe their problem-solving processes and identify evidence relevant to mental imagery. The findings are restricted to these three cases and do not establish that Visual Imagery causes differences in performance or represents the characteristics of all students within the corresponding categories.

II. MATERIAL AND METHOD

1. METHOD

This study uses a qualitative descriptive approach to gain an in-depth understanding of students' thinking processes in solving mathematical problems, particularly in the context of local culture-based learning. This approach allows researchers to comprehensively examine phenomena based on the experiences and processes of the research subjects [22, 23].

The participants were eighth-grade students at MTs Manbaul Huda Grobogan. Visual imagery was assessed using a 14-item questionnaire with four response options. Scoring and interpretation were informed by general measurement principles discussed by Azwar [24]. Each item was scored from 1 to 4, with negatively worded Items 4, 8, and 14 reverse-scored before summation. Possible total scores ranged from 14 to 56. For participant screening, this study defined categories using the empirical mean and sample standard deviation of the 31 students' total scores. The mean was calculated as $M = \Sigma X_i/N$, and the sample standard deviation as $SD = \sqrt{[\Sigma(Xi - M)^2/(N-1)]}$, yielding $M = 39{,}742$ and $SD = 5{,}615$. Categories were defined as high $(X \geq M + SD)$, moderate $(M - SD \leq X < M + SD)$, and low $(X < M - SD)$. The score thresholds were X ≥ 45,357 for the high category, 34,127 ≤ X < 45,357 for the moderate category, and X < 34,127 for the low category. These categories indicate relative questionnaire scores within the screening group.

Table 1. Visual Imagery Categorization

Score Criteria	Category	Number of students
X ≥ 45,357	High	3
34,127 ≤ X < 45,357	Moderate	25
X < 34,127	Low	3
	Total	31

Before the geometry task was administered, all 31 students in one eighth-grade class at MTs Manbaul Huda Grobogan completed the mathematical visual imagery questionnaire. Their questionnaire scores were used to classify them into high, moderate, and low visual imagery categories. One student from each category was then purposively selected, taking into account the mathematics teacher's recommendation regarding the student's ability to communicate their reasoning during an interview. The selected students were designated H1 (high visual imagery; score = 49), M1 (moderate visual imagery; score = 42), and L1 (low visual imagery; score = 29). Prior mathematics achievement records were not available for the three selected students; therefore, differences in prior attainment could not be examined as an alternative explanation for differences in task performance. Visual imagery questionnaire scores determined category membership, whereas teacher recommendations informed the selection of an individual within each category.

The instruments comprised a mathematical visual imagery questionnaire, a contextual geometry problem-solving instrument, and a semi-structured interview guide. The problem-solving instrument contained three open-ended tasks on the Pythagorean theorem; all administered during the pilot study. The mathematical tasks and visual imagery questionnaire underwent internal expert review by two members of the author team, Y. L. Sukestiyarno and Emi Pujiastuti. The review considered alignment with the intended indicators, clarity of language, and suitability for eighth-grade students. Revisions were made in response to the reviewers' feedback. Questionnaire scores were used to assign visual imagery categories, whereas written diagrams, calculations, and interview accounts were used to examine the selected students' problem-solving processes. These qualitative data were not used to determine or revise category membership.

Each subquestion was scored from 0 to 4 using an analytical rubric, giving a maximum of 20 points per task and 60 points across the three pilot tasks. The main study used only the first task, with a maximum score of 20. These task scores assessed mathematical problem-solving performance and were not used to assign visual imagery categories.

Only the first task, concerning a house roof, was administered and analyzed in the main study. It was selected after the pilot study and before main-study data collection because it required students to decompose an isosceles triangle into right triangles and identify the half-base. This offered an opportunity to examine connections among contextual information, diagrams, geometric relationships, and calculations. Because the prompt explicitly requested a right-triangle representation and the Pythagorean theorem, students' reasoning was examined within this guided structure. The other two tasks were not administered in the main study and therefore could not corroborate the three case findings.

The roof task was set in Desa Tanggungharjo, Grobogan, and described a carpenter constructing a simple house-roof frame. The geometric feature used was the isosceles-triangle profile specified in the task, rather than a documented feature of a particular Grobogan architectural tradition. Its local relevance lay in connecting the geometry problem to a named village and an everyday construction activity. However, the study did not assess whether the students regarded this setting as culturally meaningful or familiar. The task was therefore treated as a

locally contextualized geometry problem, with no claim that its roof structure was culturally distinctive to Grobogan.

In Roof-Related Geometry Task, Mr. Harto is a carpenter in Desa Tanggungharjo, Grobogan. He is constructing the frame of a simple house roof. Viewed from the side, the roof forms an isosceles triangle. The vertical height from the midpoint of the base to the apex is 3 metres, and the base is 8 metres long. Mr. Harto wants to install a wooden support extending from one end of the base to the apex, but he cannot measure its length directly. Answer the following questions:

(a) What information can you obtain from the problem? State all the known information and what needs to be determined.
(b) What strategy would you use to solve the problem? Explain your planned steps and why you chose that strategy.
(c) Draw a fully labelled visual representation of the roof situation using a right-triangle model, and write the corresponding equation using the Pythagorean theorem.
(d) Solve the problem using your model. Calculate the length of the wooden support and state your conclusion in the context of the problem.
(e) Check your answer. If the base is increased to 12 metres while the height remains 3 metres, how would the length of the wooden support change? Explain mathematically.

Before the main study, the instruments were piloted with 31 eighth-grade students who were not included in the participant-screening sample. The visual imagery questionnaire yielded a Cronbach's alpha coefficient of 0.761. For the problem-solving instrument, Cronbach's alpha was 0.830 across the 15 scored subparts of the three pilot tasks. These coefficients describe internal consistency in the pilot sample.

Data collection comprised questionnaire administration, a written geometry task, and follow-up interviews. All 31 students completed the visual imagery questionnaire and subsequently undertook the written task. The roof-related task was administered once, with an allocated completion time of 90 minutes. No diagram was supplied; the diagrams in students' responses were student-generated. However, subquestion (c) explicitly requested a right-triangle representation and the application of the Pythagorean theorem. Students' representations and strategy use were therefore interpreted in relation to this guidance. Written responses from three selected students, one from each visual imagery category, were analyzed in depth. No calculators, rulers, notes, or other aids were used during task completion, and no assistance was provided. The three selected students were subsequently interviewed individually in Indonesian by N. N. F. Auliya. The semi-structured interviews were audio-recorded, and students could refer to their written responses while explaining their reasoning. They were not asked to revise their written answers.

Data were analyzed qualitatively using an interactive analysis model [25], including data reduction, data presentation, and conclusion drawing. N. N. F. Auliya analyzed the written responses and audio-recorded interviews using the problem-solving stages and imagery indicators as analytical categories. The interpretations were reviewed by the other co-authors. Each case was examined individually before similarities and differences across the three cases were identified.

The analysis considered image formation, maintenance, and transformation. Image formation was examined through students' descriptions of the roof and its geometric elements; image maintenance through their explanations of how they retained relevant spatial relationships while solving the problem; and image transformation through their explanations of dividing or reconfiguring the roof structure into a right-triangle model. For example, H1's retrospective description of a triangular roof with a central height was coded as a description of an imagined roof structure and considered alongside the written diagram as evidence relevant to image formation. Written diagrams and calculations were compared with retrospective interview explanations to identify consistencies and discrepancies. A correct diagram alone was not treated as sufficient evidence of mental imagery. Interpretations also considered alternative explanations, including students' geometric knowledge and the guidance provided in the task. Member checking of the researchers' interpretations was not conducted.

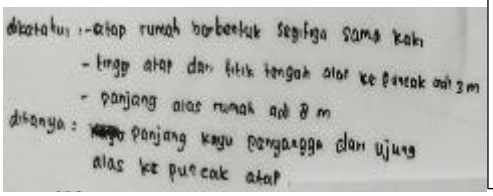
2. *ETHICAL CONSIDERATIONS*

Verbal permission for the use of students' written responses in research publications was obtained from the students and the school. Parent or guardian consent and formal ethics committee approval were not obtained. Participants are identified using the codes H1, M1, and L1.

III. RESULTS AND DISCUSSION

1. STUDENTS WITH HIGH VISUAL IMAGERY

1.1. Understanding the Problem



Known:
- The roof of the house is in the shape of an isosceles triangle.
- The height of the roof from the midpoint of the base to the peak is 3 m.
- The length of the base of the house is 8 m.

Asked:
Determine the length of the supporting wood from the end of

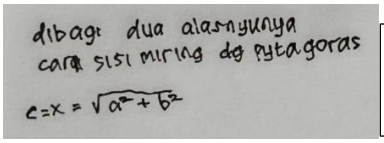
FIGURE 1. Mental Representation of Roof Structure by High VI Students

The student with high VI identified the known information and the required length. The student recognized that the roof formed an isosceles triangle with a height of 3 metres and identified the supporting timber as the hypotenuse of the right-triangle model. In the retrospective interview, the student described imagining a triangular roof with a central height line. The following excerpt illustrates this explanation.

Q : How do you imagine the shape of the roof of the house in the question?
H1 : "I imagine it like a normal house roof that is triangular in shape with a high line in the middle."
Q : Why do you know that what you are looking for is the hypotenuse of the triangle?
H1 : "Because the supporting timber is positioned at an angle from the bottom to the top of the roof."

The description of an imagined triangular roof provides retrospective verbal evidence relevant to image formation. The explanation of the supporting timber's position also indicates recognition of a geometric relationship. However, these statements do not establish how the student inspected or maintained a mental image during task completion, or whether the imagined configuration preceded the written drawing.

1.2. Planning Strategy



The base is divided into two equal parts.
Use the Pythagorean theorem to find the slanted side."
$c = x = \sqrt{a^2 + b^2}$

FIGURE 2. Transformation of an Isosceles Triangle into a Right Triangle

The student with high VI planned to use the Pythagorean theorem by dividing the isosceles triangle along its height and using half of the base. This approach was consistent with the geometric relationships required for the solution. In the interview, the student explained why the resulting right triangle and equal base segments were relevant.

Q : Why did you choose to use the Pythagorean Theorem?
H1 : "Because after dividing it, there is a right triangle and what we are looking for is the hypotenuse."
Q : Why is the base of the triangle divided into two parts?
H1 : "Because the altitude of an isosceles triangle divides the base into two equal lengths."

These explanations provide evidence of the student's knowledge of the relationship between the altitude and base of an isosceles triangle. They support the interpretation that the student understood the right-triangle model used in the solution. However, the excerpt does not specify whether the division was performed mentally or identified through the written diagram. It therefore does not independently establish image transformation. The strategy must also be interpreted in relation to the task's explicit request for a right-triangle representation and the Pythagorean theorem.

1.3. Mathematical Modeling

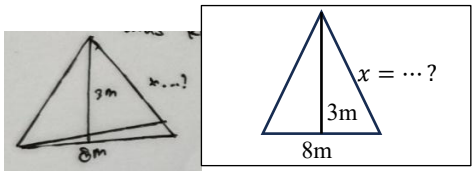


FIGURE 3. Representational Conversion to Geometric and Symbolic Models

The student with high VI represented the verbal information using a roof diagram and a corresponding symbolic model. The student labelled the relevant measurements and identified the required length as the hypotenuse of the right triangle. The interview explanation described how the drawing helped identify the known and unknown sides.

Q : How do you turn a story problem into a triangle picture?
H1 : "I draw a triangle according to the shape of the roof of the house and then measure the height and base according to the question."
Q : Did the drawings you created help determine the solution?
H1 : "Yes, because from the picture I know the known side and the side I am looking for."

The written work and interview account indicate coordination between the problem description and the external geometric representation. The student explicitly referred to the drawing as support for identifying the relevant sides. This provides evidence of diagram use and geometric understanding, but does not by itself establish that the student converted a previously formed mental image into the drawing or maintained that image while constructing the symbolic model.

1.4. Implementing the Solution

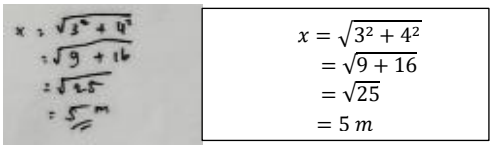


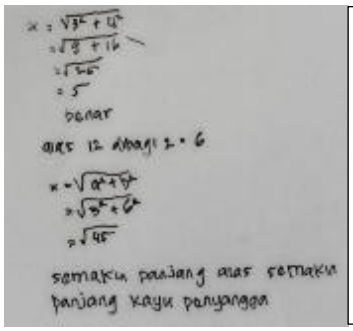
FIGURE 4. Systematic Implementation Using the Pythagorean Theorem

The student with high VI carried out the calculation using the previously constructed right-triangle model. The student first identified half of the 8-metre base as 4 metres, then applied the Pythagorean theorem with the height of 3 metres to obtain the correct supporting-wood length of 5 metres. In the interview, the student described this sequence and referred to the formula and the created image when explaining the result.

Q : What calculation steps did you take?
H1 : "First, I divided the 8-meter base in half to make 4 meters, then calculated using the Pythagorean formula."
Q : How do you know that your calculation results are correct?
H1 : "Because the results match the formula and match the image I created."

The calculation and interview explanation were consistent in their use of the half-base and the Pythagorean theorem. This supports the interpretation that the student connected the geometric model with the numerical procedure. However, the reference to "the image I created" does not clarify whether the student meant the written diagram or an internal mental image. The available evidence therefore supports a correct geometric procedure, without establishing that stable mental imagery was maintained throughout the calculation.

1.5. Evaluation and Reflection



$x = \sqrt{3^2 + 4^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$
Correct.
The base 12is divided by 2, resulting in 6.
$x = \sqrt{a^2 + b^2}$
$= \sqrt{3^2 + 6^2}$
$= \sqrt{45}$
"The longer the base, the longer the supporting wood."

FIGURE 5. Verification of Final Answers by the Student with High VI

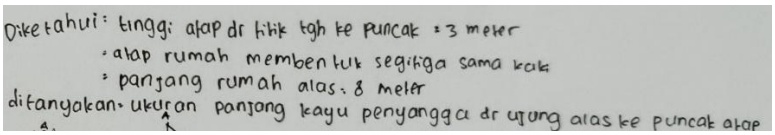
The student with high VI checked the original calculation and marked the result of 5 metres as correct. Figure 5 also shows a response to the modified condition in which the base increased to 12 metres while the height remained 3 metres. The student identified the new half-base as 6 metres, calculated the supporting-wood length as √45 metres, and concluded that a longer base would require a longer supporting piece of wood. In the interview, the student explained both the recalculation and the reasonableness of the original result in the roof context. The following excerpt illustrates the student's explanation:

Q : When you read the question, did you immediately imagine the shape of the roof of the house?
M1 : "Yes, I imagine the roof of the house is triangular."
Q : Is there any part of the question that confuses you?
M1 : "At first I was confused about which part of the wooden support was which."

The written response and interview provide evidence of checking the calculation, assessing contextual reasonableness, and applying the geometric model to a changed condition. The comparison between the supporting-wood length and the roof's height indicates attention to a geometric relationship, while the modified-base calculation shows consideration of how changing one measurement affects the solution. These observations do not support describing the student's reflection as merely arithmetic checking or broadly weak. Although the interview excerpt does not include an explicit comparison of alternative strategies, this absence does not establish an inability to evaluate such strategies. The available evidence does not clarify whether the checks involved a mental image, the written diagram, or both.

2. STUDENTS WITH MODERATE VISUAL IMAGERY

2.1. Understanding the Problem



Known:
- The height of the roof from the midpoint to the peak is 3 meters.
- The roof of the house forms an isosceles triangle.
- The length of the house base is 8 meters.

Asked:
- Determine the length of the supporting wood from the end of the base to the peak of

FIGURE 6. Mental Representation of Roof Structure by the Student with Moderate VI

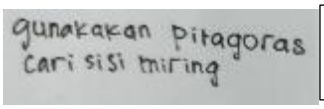
The student with moderate VI identified the roof as an isosceles triangle with a height of 3 metres and a base length of 8 metres. In the retrospective interview, the student described imagining a triangular roof but reported initial confusion about the position of the supporting timber. The following excerpt illustrates this explanation.

Q : When you read the question, did you immediately imagine the shape of the roof of the house?

M1 : "Yes, I imagine the roof of the house is triangular."
Q : Is there any part of the question that confuses you?
M1 : "At first I was confused about which part of the wooden support was which."

The description of an imagined triangular roof provides retrospective verbal evidence relevant to image formation. However, imagining the overall shape did not necessarily mean that the student had identified the required side. The reported confusion concerns the interpretation of a geometric element; it does not independently establish instability in image maintenance or inspection.

2.2. Planning Strategy



"Use the Pythagorean theorem to find the hypotenuse."

FIGURE 7. Strategic Planning Based on Simple Geometric Representation

The student with moderate VI selected the Pythagorean theorem and reported reviewing the diagram when uncertain about the strategy. The explanation initially referred to familiarity with the formula rather than explicitly identifying the geometric conditions for its application.

Q : Why did you choose to use the Pythagorean Theorem?
M1: "Because usually when you find the side of a triangle you use that formula."
Q : Are you immediately convinced by the strategy used?
M1: "I wasn't immediately sure; I took another look at the picture."

These statements indicate that the student associated triangle problems with the Pythagorean theorem and used the external diagram to reconsider the planned approach. Reviewing the diagram may have supported the student's reasoning, but this action alone does not establish how a mental image was maintained or inspected. The strategy choice must also be interpreted in relation to the task's explicit request to use the Pythagorean theorem.

2.3. Mathematical Modeling

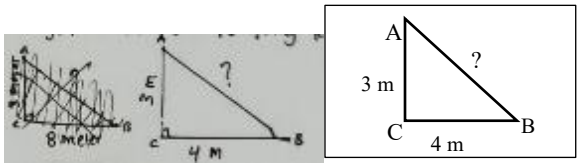


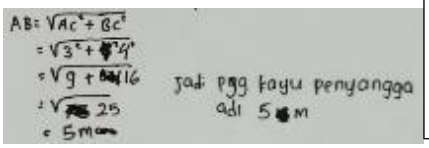
FIGURE 8. Geometric Model Transformation with Initial Error in Determining the Base

The student with moderate VI drew a right-triangle model but initially assigned the full base length of 8 metres to its horizontal side. The student subsequently identified the relevant length as 4 metres. In the retrospective interview, the student attributed this correction to reviewing the diagram and recognizing that the height divided the original base into two equal parts.

Q : Why did you use a base of 8 meters in a right triangle?
M1: "Because I immediately took the base number in the question."
Q : Why did you finally realize that the base was not 8 meters but 4 meters?
M1: "Because I saw again that the height line divides the base of the triangle into two equal lengths."

The explanation indicates recognition of the relationship between the altitude and half-base after reviewing the diagram. This provides evidence of a correction in geometric reasoning and suggests a useful role for the external representation in this case. However, the account does not establish whether the student mentally transformed the roof configuration, reinterpreted the written diagram, or combined both processes.

2.4. Implementing the Solution



$$AB = \sqrt{AC^2 + BC^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5 \text{ m}$$

"So, the length of the supporting wood is 5 meters."

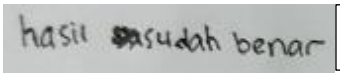
FIGURE 9. Solution Process with Base-Length Correction by the Student with Moderate VI

The student with moderate VI obtained the correct supporting-wood length of 5 metres after replacing the full base of 8 metres with the half-base of 4 metres in the right-triangle calculation. The student applied the Pythagorean theorem using the corrected measurement and the height of 3 metres. The interview explanation linked this correction to another examination of the diagram.

Q : Have you ever been confused when doing calculations?
M1 : "Yes, at first I was confused because I previously used an 8-meter base."
Q : How do you know that the calculation using a base of 8 meters is wrong?
M1 : "Because after looking at the picture of the triangle again, it turns out that the base of the right triangle is only half of the original base."

The written solution and interview account indicate that identifying the half-base allowed the student to apply the numerical procedure appropriately. The student's explanation suggests that reviewing the diagram supported the connection between the geometric relationship and the calculation. This use of an external representation should not itself be interpreted as evidence of unstable imagery. The available account does not establish how the student maintained or manipulated a mental image during the calculation.

2.5. Evaluation and Reflection



"The result is already correct."

FIGURE 10. Limited Evaluation and Reflection by the Student with Moderate VI

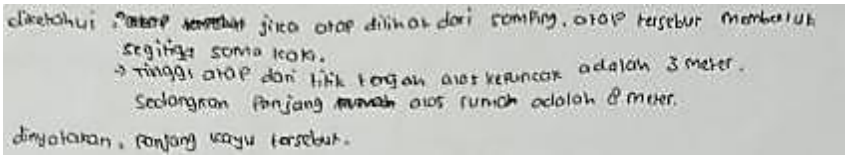
The student with moderate VI marked the result as correct in the written response. In the interview, the student justified this judgment by referring to the formula but expressed uncertainty about how increasing the base would affect the hypotenuse. The following excerpt illustrates these explanations.

Q : How do you make sure your answer is correct?
M1 : "Because the calculation results are in accordance with the formula."
Q : If the length of the base of the triangle is increased, what do you think will happen to the length of the hypotenuse?
M1 : "I don't really know because I'm confused about how it will affect the calculation results."

The written statement and interview provide evidence of a formula-based justification of the answer. However, the interview explanation did not establish how the student evaluated the geometric relationship under the changed-base condition. This uncertainty may relate to geometric knowledge or difficulty applying the procedure to a modified situation; it does not independently demonstrate inconsistent image inspection or an inability to use mental imagery. No explicit comparison of alternative solution strategies was evident in the quoted responses.

3. STUDENTS WITH LOW VISUAL IMAGERY

3.1. Understanding the Problem



Known:
- When viewed from the side, the roof forms an isosceles triangle.
- The height of the roof from the midpoint of the base to the peak is 3 meters.
- Meanwhile, the length of the house base is 8 meters.

Asked:

FIGURE 11. Understanding of Problems Focusing on Numerical Information by the Student with Low VI

The student with low VI identified the measurements of 3 metres and 8 metres. However, the interview explanation did not clarify how these measurements related to the required supporting-wood length. When asked about the relationships between the triangle's elements, the student reported not yet understanding them.

Q : What information do you know from the question?
L1 : "There are 3 meters and 8 meters long."
Q : Do you understand the relationship between the height, base, and sides of the triangle in the question?
L1 : "Not yet."

These responses indicate that the student identified numerical information but did not explain the relevant geometric relationships in this excerpt. This suggests a difficulty in interpreting the problem's geometric structure. However, the responses do not establish whether the student formed a mental image of the roof. Difficulty explaining the relationships may also reflect limitations in geometric knowledge or in articulating that knowledge during the interview.

3.2. Planning Strategy

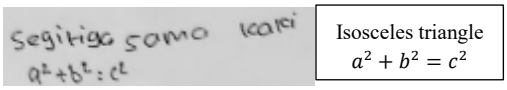


FIGURE 12. Procedural Strategy Based on Memorization of Formulas by the Student with Low VI

The student with low VI selected the Pythagorean theorem but did not explain how the roof configuration met the conditions for its application. Instead, the student referred to an association between triangle problems and the formula and reported wanting to proceed directly to calculation.

Q : Why did you choose to use the Pythagorean theorem?
L1 : "Because triangle problems usually use that formula."
Q : Did you think about the shape of the triangle before calculating?
L1 : "Not really, I want to count straight away."

The explanation suggests that the student selected a familiar formula without articulating the right-triangle relationships needed for its application. The strategy choice must also be interpreted in relation to the task's explicit request to use the Pythagorean theorem. Although the student reported giving little attention to the shape before calculating, this statement does not establish an inability to form, inspect, or transform mental images. The evidence primarily concerns the justification of the strategy used in this task.

3.3. Mathematical Modeling

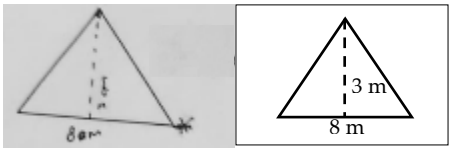


FIGURE 13. Incomplete Geometric Representation by the Student with Low VI

The student with low VI drew a representation of the roof but used the full base length of 8 metres in the calculation rather than identifying the half-base required for the right-triangle model. In the interview, the student explained that this measurement was taken directly from the problem and reported uncertainty about which part of the drawing to use.

Q : Why do you still use the 8-meter base in the calculations?
L1 : "Because the length of the base in the question is 8 meters."
Q : Did the drawings you created help determine the solution?
L1 : "It helps a little, but I'm still confused about which part to use for counting."

The written work and interview account indicate difficulty connecting the roof representation with the measurements required for the right-triangle model. Although the student produced a drawing, the half-base relationship was not incorporated into the calculation. This suggests difficulty applying the relevant geometric properties, rather than establishing a failure of mental image transformation. The available explanation does not clarify whether the student mentally divided the roof configuration or attempted to interpret the relationship solely through the written diagram.

3.4. Implementing the Solution

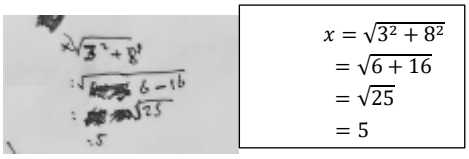


FIGURE 14. Mechanical Calculation Without Base Transformation

The student with low VI obtained a final answer of 5 metres, which is the correct length of the supporting wood. However, the written procedure did not support this answer. The student directly used the full base length of 8 metres and the height of 3 metres in the Pythagorean theorem, without first identifying half of the base as a side of the right triangle. The student also wrote 3^2 as 6 and 8^2 as 16, then proceeded to $\sqrt{25}$, although 6 + 16 equals 22. Thus, while the final numerical answer was correct, the geometric setup and calculations shown were incorrect. The following interview excerpt provides further insight into the student's approach:

Q : What calculation steps did you take?
L1 : "I immediately entered the numbers 3 meters and 8 meters into the Pythagorean formula."
Q : Have you ever thought that the base of the triangle needs to be divided in half first?
L1 : "No, I just calculated using the numbers in the question."

The interview account was consistent with the written use of the measurements stated in the problem. The response to the second question was elicited by an explicit prompt about dividing the base and should therefore be interpreted as a retrospective explanation of the written procedure. The geometric modeling error and the errors in squaring and addition represent distinct difficulties; neither should be attributed exclusively to visual imagery. The available evidence does not establish how the student arrived at 5 metres through the incorrect intermediate steps or which mental imagery processes were involved.

3.5. Evaluation and Reflection

No written responses were given by low VI students for the evaluation and reflection indicators.

In the interview, the student with low VI reported submitting the solution without checking it. When asked how increasing the base would affect the hypotenuse, the student did not provide an explanation.

Q : Have you checked the answers you have obtained?
L1 : "No, I collected the answers right away."
Q : If the length of the base of a triangle is increased, what do you think will happen to the length of the hypotenuse?
L1 : "I don't know."

The absence of a written response and the reported lack of checking indicate that verification was not demonstrated in the available evidence. The response to the changed-base question also provided no explanation of the relevant geometric relationship. These observations do not establish an inability to form, inspect, or use mental images. Geometric knowledge, the calculation difficulties identified in the written solution, and willingness or ability to articulate an explanation remain possible considerations when interpreting this case.

IV. DISCUSSION

1. SYNTHESIS OF THINKING PROCESS CHARACTERISTICS BASED ON VISUAL IMAGERY

The following table summarizes the problem-solving processes of H1, M1, and L1, who were selected from the high, moderate, and low visual imagery categories, respectively. The comparison is based on their written responses and retrospective interview explanations for the roof-related task.

TABLE 2. Synthesis of Characteristics of Students' Thinking Processes Based on Visual Imagery Categories

Category VI	Understanding the Problem	Planning Strategy	Mathematical Modeling	Implementing the Solution	Evaluation & Reflection	General Characteristics
High VI	Build a complete and fast mental image	Flexible strategies based on geometric relationships	Accurately convert mental images into geometric and symbolic models	Systematic with image-based reasoning	Verifying results; reflection not yet in-depth	Conceptual, stable, integrated
VI Medium	Understanding the context; the mental image is not yet fully stable	Adequate strategy but relies on external visual aids	Geometric model is adequate; image transformation is inconsistent.	Calculations are relatively correct; still procedural at some stages	Limited evaluation; relies on external visuals	Semi-conceptual with evolving imagery
Low VI	Difficulty building a mental image of a triangular relationship	Formula memorization-based strategy	Incomplete geometric model; weak representational conversion	Mechanical solutions without adequate image-based reasoning	There is almost no reflection or inspection of mental images.	Procedural with Visual Imagery is very limited

In the roof-related task, the three selected students differed in how they connected contextual information, geometric representations, and calculations. The student with high VI used the height and half of the base in the right-triangle model to calculate the supporting-wood length. The student with moderate VI initially used the full base of 8 metres but subsequently identified the half-base as 4 metres. In the retrospective interview, this student attributed the correction to reviewing the diagram and recognizing that the height divided the base into two equal parts. This account suggests that the diagram supported the correction in this particular case, although its contribution cannot be separated from other possible influences.

The student with low VI used the full base of 8 metres in the calculation. Although the final numerical answer of 5 metres was correct, the geometric setup and arithmetic shown did not support that result. Regarding evaluation and reflection, the student with high VI reported rechecking the calculation and assessing whether the answer was reasonable in context. The student with moderate VI referred to the formula when checking the answer but had difficulty explaining the effect of an increased base. The student with low VI reported not checking the answer. These findings describe the three individual cases on this task and do not establish patterns applicable to other students or to their respective visual imagery categories.

2. VISUAL IMAGERY AND COGNITIVE PROCESSES IN MATHEMATICAL PROBLEM SOLVING

The three cases provide insight into how the selected students coordinated geometric representations and calculations in the roof-related task. The student with high VI identified the half-base and used it with the roof's height to calculate the supporting-wood length. The correspondence between the written model and the interview explanation suggests an understanding of the relevant geometric relationships in this case. However, it does not independently establish that a mental image was maintained throughout the solution process. The distinction between internal representations and observable external visualizations is important when interpreting this evidence [14]. Prior geometric knowledge and the task's explicit request for a right-triangle representation and the Pythagorean theorem also provide possible explanations for the approach taken.

The student with moderate VI initially used the full base of 8 metres before correcting it to 4 metres. In the retrospective interview, the student attributed this correction to reviewing the diagram and recognizing that the height divided the base into two equal parts. This account suggests that the external representation supported the identification and correction of a geometric relationship in this case. This interpretation is consistent with research

identifying positive effects of external visualization interventions on mathematics learning [9]. However, the present study did not test a visualization intervention, and the correction cannot be attributed to the diagram alone. Diagram use should therefore not be interpreted solely as evidence of weak or unstable imagery. It may also reflect a productive interaction between an external representation and the student's geometric reasoning [14]. The available evidence does not establish whether the correction involved a transformation of a mental image, a reinterpretation of the drawn figure, or both.

The student with low VI directly used the full base of 8 metres and made errors in squaring and addition, yet wrote the correct final answer of 5 metres. This discrepancy distinguishes numerical correctness from a mathematically supported solution. Attention to the procedure and reasoning shown in written solutions is also reflected in Docktor et al.'s assessment framework, although that framework was developed for introductory physics [26]. The interview account suggests that the student selected the measurements directly from the problem without identifying the half-base required for the right-triangle model. Nevertheless, these difficulties cannot be attributed exclusively to visual imagery: knowledge of geometric properties, understanding of exponentiation, and arithmetic proficiency are also relevant explanations. The written errors alone do not establish a failure to form or transform a mental image.

Taken together, the cases illustrate that numerical accuracy, geometric understanding, and imagery-related explanations need not coincide. The low-VI case shows that a correct answer can conceal an invalid procedure, whereas the moderate-VI case shows that reviewing an external diagram can accompany a successful correction rather than simply indicate deficient imagery. The high-VI case provides evidence of an appropriate geometric procedure, but does not establish continuous maintenance of a mental image. The added understanding is therefore a more differentiated interpretation of solution evidence: neither questionnaire category, answer accuracy, nor diagram use alone adequately characterizes the imagery processes involved in these cases. Research identifying different relationships between visual and spatial working memory and specific mathematical competencies further cautions against treating these constructs as interchangeable [27]. Because drawing ability and spatial knowledge were not separately assessed, and prior mathematics achievement records were unavailable, the study cannot isolate the contribution of visual imagery to the observed differences. The interpretations remain restricted to these three students and this guided task.

3. EVALUATION AND REFLECTION REMAIN A CHALLENGE

The three students differed in the evaluation and reflection demonstrated in their responses. The student with high VI reported recalculating the solution and assessing its reasonableness by explaining that the supporting wood should be longer than the roof's height. This explanation extends beyond checking arithmetic to considering a geometric relationship in context. However, the interview excerpt did not include an explicit comparison of alternative strategies or an evaluation of their effectiveness. The distinction between checking a result and evaluating a strategy is relevant to the broader discussion of metacognitive monitoring and regulation [10].

The student with moderate VI referred to consistency with the formula when explaining confidence in the answer but had difficulty explaining how an increase in the base would affect the supporting-wood length. Nevertheless, the earlier correction from 8 metres to 4 metres, which the student attributed to reviewing the diagram, provides evidence of reconsidering a geometric relationship during the solution process. This correction should be distinguished from the student's limited explanation of the changed condition. The student with low VI provided no written response to the evaluation item and reported submitting the answer without checking it. These observations describe the evaluation expressed in the available written responses and retrospective interviews; they do not establish the students' general reflective abilities.

These cases suggest the value of distinguishing arithmetic checking, contextual reasonableness, correction of a geometric relationship, and evaluation of alternative strategies when examining reflection. Susanna et al. [28] found that metacognitive skills often required reflective prompts to become evident during mathematical problem solving and interviews. Their findings highlight the importance of considering how interview questions elicit reflective explanations, rather than treating the absence of an unprompted explanation as evidence that a student lacks reflective ability. They also indicate why a student's visual imagery questionnaire category alone cannot explain the evaluation demonstrated on this task. Research on metacognitive monitoring and mathematical self-assessment provides a basis for investigating prompts that encourage students to justify their procedures and assess their solutions [10,29]. Such prompts remain a potential instructional implication, as their effectiveness was not tested in the present study.

4. IMPLICATIONS FOR MATHEMATICS LEARNING

The three cases suggest possible directions for investigating geometry instruction. The moderate-VI student's explanation linked reviewing the diagram with correcting the full base of 8 metres to the half-base of 4 metres. In contrast, the low-VI student produced a roof drawing but did not use the half-base in the calculation. These observations suggest that instructional attention could extend beyond producing diagrams to interpreting the geometric relationships they represent. Possible activities include marking the height and half-base, identifying the relevant right triangle, and explaining how its measurements correspond to a symbolic equation. Such activities are consistent with the distinction between constructing and interpreting external visualizations [14], but their effectiveness was not examined in this study.

The findings also suggest investigating how diagram review might support connections between geometric models and calculations. For example, learners could be asked to explain which segment each numerical value represents before substituting it into a formula. Support could then be adjusted according to the difficulties demonstrated in their responses, rather than prescribed solely on the basis of a visual imagery questionnaire category. Although previous intervention research has identified benefits of external visualizations for mathematics learning [9], the present cases do not establish which form or amount of visual support would be effective for these students.

Evaluation activities could also distinguish checking arithmetic, judging contextual reasonableness, examining geometric relationships, and considering alternative strategies. The student with high VI reported both recalculation and a contextual judgment that the supporting wood should be longer than the roof's height. The student with moderate VI described correcting a geometric relationship but had difficulty explaining the changed-base condition, whereas the student with low VI reported not checking the answer. These observations provide a rationale for investigating prompts such as "Why is this measurement appropriate?", "Does the result fit the roof situation?", and "How would the solution change if the base increased?" Research on reflective prompts and mathematical self-assessment offers relevant perspectives for developing such activities [28, 29]. However, this study did not test whether these prompts improve reflection or mental imagery.

Future research could compare these instructional approaches across additional students and geometry tasks while separately assessing geometric knowledge, drawing ability, and arithmetic proficiency. It could also document when corrections occur and whether they follow independent diagram review or interviewer guidance. Such evidence would help clarify how external representations and students reported mental images contribute to solution revision, without assuming that the differences observed in these three cases apply to all students within the corresponding visual imagery categories.

5. RESEARCH LIMITATIONS

This study examined three individual cases on a single contextual geometry task. The findings therefore describe these students' responses under the study conditions and should not be generalized to all students within the corresponding visual imagery categories. Prior mathematics achievement records were unavailable, preventing examination of previous mathematics attainment as an alternative explanation for differences in performance. Drawing ability and spatial knowledge were not assessed independently, and general mathematics ability was not measured separately. Consequently, the study could not isolate the contribution of visual imagery from these related competencies when interpreting the three students' diagrams, calculations, and explanations. Selection also considered teacher recommendations regarding verbal communication ability, potentially favoring students who could articulate their reasoning more readily.

The task incorporated a local setting and an everyday construction activity, but its architectural distinctiveness and cultural meaningfulness to the participating students were not established. Although no diagram was supplied, the task explicitly requested a right-triangle representation and the application of the Pythagorean theorem. Students' strategy use should therefore be interpreted in relation to this guidance rather than as entirely spontaneous strategy selection. The interviews elicited retrospective explanations of students' written solutions, which may not fully capture their thinking during task completion. Visual scaffolding remains a potential instructional implication, not an intervention whose effectiveness was established by this study. Future research could examine these patterns across more participants and tasks, incorporate prior achievement data, and directly investigate the effects of visual scaffolding.

V. CONCLUSION

This study provides a case-based account of how three selected students from different visual imagery questionnaire categories coordinated representations and calculations when solving a single roof-related geometry task. The student with high VI used the height and half-base in an appropriate right-triangle model. The student with moderate VI initially used the full base but subsequently corrected it, attributing this correction to reviewing the diagram. The student with low VI wrote the correct numerical answer of 5 metres, although the geometric setup and arithmetic did not support that result. These cases illustrate the importance of examining written procedures alongside students' explanations rather than inferring understanding or mental imagery from the final answer alone.

Evaluation and reflection differed among the three students. The student with high VI checked the calculation, considered the contextual reasonableness of the result, and applied the geometric model to the increased-base condition. The available interview excerpt did not include an explicit comparison of alternative strategies. The student with moderate VI referred to the formula when checking the answer but had difficulty explaining the effect of an increased base. The student with low VI reported not checking the answer. These observations indicate limitations in the evaluation demonstrated in the available responses, rather than establishing evaluation and reflection as the weakest aspects of all visual imagery categories.

The study contributes an empirical illustration of why correct answers, diagram use, and students' reports of mental images require separate but coordinated interpretation when examining imagery-related reasoning. However, it does not establish that visual imagery caused the observed differences or isolate its contribution from geometric knowledge, drawing ability, and arithmetic proficiency. The findings are restricted to the three students and the guided task examined. Diagram review and reflection on geometric relationships warrant further investigation as possible instructional supports; their effectiveness was not tested in this study.

Conflicts of Interest

The authors declare no conflicts of interest.

Author Contributions

Conceptualization, N.N.F.A.; methodology, N.N.F.A., S.M., and B.E.S.; software, N.N.F.A.; validation, Y.L.S. and E.P.; formal analysis, N.N.F.A. and B.E.S.; investigation, N.N.F.A.; resources, N.N.F.A.; data curation, N.N.F.A.; writing—original draft preparation, N.N.F.A.; writing—review and editing, N.N.F.A., S.M., B.E.S., Y.L.S., and E.P.; visualization, N.N.F.A. and B.E.S.; supervision, Y.L.S. and E.P.; project administration, N.N.F.A.; funding acquisition, N.N.F.A. All authors have read and agreed to the published version of the manuscript.

Funding Statement

This study has no external funding.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments

The authors would like to express their sincere gratitude to all participants who contributed to this study and to the reviewers for their valuable feedback. This research was funded by the authors without external research funding.

REFERENCES

1. OECD. (**2023**). *PISA 2022 results (Volume I): The state of learning and equity in education*. OECD Publishing.
2. Schoenfeld, A. H., & Sloane, A. H. (Eds.). (**2016**). *Mathematical thinking and problem solving*. Routledge.
3. Pólya, G. (**1945**). *How to solve it: A new aspect of mathematical method*. Princeton University Press.
4. Nguyen, H. A., Guo, Y., Stamper, J. C., & McLaren, B. M. (**2020**). Improving students' problem-solving flexibility in non-routine mathematics. In I. I. Bittencourt, M. Cukurova, K. Muldner, R. Luckin, & E. Millán (Eds.), *Artificial intelligence in education* (Lecture Notes in Computer Science, Vol. 12164, pp. 409–413). Springer.

5. Sánchez-Barbero, B., Chamoso, J. M., Vicente, S., & Rosales, J. (**2020**). Analysis of teacher-student interaction in the joint solving of non-routine problems in primary education classrooms. *Sustainability, 12*(24), Article 10428.
6. Ruamba, M. Y., Sukestiyarno, Y. L., Rochmad, R., & Asih, T. S. N. (**2025**). The impact of visual and multimodal representations in mathematics on cognitive load and problem-solving skills. *International Journal of Advanced and Applied Sciences, 12*(4), 164–172.
7. National Council of Teachers of Mathematics. (**2000**). *Principles and standards for school mathematics*.
8. OECD. (**2023**). PISA 2022 mathematics framework. In *PISA 2022 assessment and analytical framework*. OECD Publishing.
9. Schoenherr, J., Strohmaier, A. R., & Schukajlow, S. (**2024**). Learning with visualizations helps: A meta-analysis of visualization interventions in mathematics education. *Educational Research Review, 45*, Article 100639.
10. Veenman, M. V. J. (**2017**). Learning to self-monitor and self-regulate. In R. E. Mayer & P. A. Alexander (Eds.), *Handbook of research on learning and instruction* (2nd ed., pp. 233–257). Routledge.
11. Kosslyn, S. M. (**1995**). Mental imagery. In S. M. Kosslyn & D. N. Osherson (Eds.), *An invitation to cognitive science: Visual cognition* (2nd ed., Vol. 2, pp. 267–295). MIT Press.
12. Presmeg, N. (**2020**). Visualization and learning in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 900–904). Springer.
13. Munawwir, Z., Susanto, S., & Suwito, A. (**2025**). Mental image and its impact on types of errors in mathematical problem solving: A case study. *ETDC: Indonesian Journal of Research and Educational Review, 5*(1), 513–524.
14. Schoenherr, J., & Schukajlow, S. (**2024**). Characterizing external visualization in mathematics education research: a scoping review. *ZDM – Mathematics Education*, *56*(1), 73–85.
15. Hegarty, M., & Kozhevnikov, M. (**1999**). Types of Visual-Spatial Representations and Mathematical Problem Solving. In *Journal of Educational Psychology*, 91(4), 684–689.
16. Arcavi, A. (**2003**). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.
17. Baddeley, A. (**2003**). Working memory: Looking back and looking forward. *Nature Reviews Neuroscience, 4*(10), 829–839.
18. Duval, R. (**1999**). *Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning* [Conference paper]. 21st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Cuernavaca, Mexico.
19. Cahyadi, W., Faradisa, M., Cayani, S., & Syafri, F. S. (**2020**). Etnomatematika untuk meningkatkan kemampuan pemecahan masalah matematis siswa. *ARITHMETIC: Academic Journal of Math, 2*(2), 157–168.
20. Iswara, H. S., Ahmadi, F., & Ary, D. D. (**2022**). Numeracy literacy skills of elementary school students through ethnomathematics-based problem solving. *Interdisciplinary Social Studies, 2*(2), 1604–1616.
21. D'Ambrosio, U. (**2008**). O Programa Etnomatemática: Uma síntese [The ethnomathematics program: A summary]. *Acta Scientiae, 10*(1), 7–16.
22. Creswell, J. W., & Poth, C. N. (**2018**). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE Publications.
23. Merriam, S. B., & Tisdell, E. J. (**2016**). *Qualitative research: A guide to design and implementation* (4th ed.). Jossey-Bass.
24. Azwar, S. (**2012**). *Reliabilitas dan validitas*. Pustaka Pelajar.
25. Miles, M. B., Huberman, A. M., & Saldaña, J. (**2014**). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications.
26. Docktor, J. L., Dornfeld, J., Frodermann, E., Heller, K., Hsu, L., Jackson, K. A., Mason, A., Ryan, Q. X., & Yang, J. (**2016**). Assessing student written problem solutions: A problem-solving rubric with application to introductory physics. *Physical Review Physics Education Research, 12*(1), Article 010130.
27. Silverman, S., & Ashkenazi, S. (**2022**). The unique role of spatial working memory for mathematics performance. *Journal of Numerical Cognition, 8*(1), 226–243.
28. Susanna, T., Lasse, E., Henrik, N. J., & Sari, H.-N. (**2026**). From Problem-Solving to Reflection: Activating Diverse Metacognitive Skills in Mathematics. *International Journal of Science and Mathematics Education*, *24*(2), 10.
29. Barana, A., Boetti, G., & Marchisio, M. (**2022**). Self-assessment in the development of mathematical problem-solving skills. *Education Sciences, 12*(2), Article 81.